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A PRIORI ESTIMATES FOR THE SOLUTION OF THE DEGENERATE THIRD ORDER DIFFERENTIAL EQUATION

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Sufficient conditions for existence of the solution and coercive estimation for the solution of a third-order linear differential equation with a variable high coefficient in the following form

$$l_\lambda y = -\sqrt{5+x^2}\left(\sqrt{5+x^2}\,y'\right)'' + [q(x)+ir(x)+\lambda]y = f(x), \qquad (1)$$

are obtained in this work, here $f \in L_p \equiv L_p(R),\ 1 \le p < +\infty,\ \ \lambda \ge 0$.

In the works of R.D.Akhmetkaliyeva, L.-E.Persson, K.N.Ospanov, P.Wall [1], which was published in 2015 various cases of the third-order linear differential equation with variable high coefficient are studied in details, and the results are presented. The fifth-order linear differential equation with a variable high coefficient was considered in the researching work of A.E.Muslim [2].

The general form

$$(L+\lambda E)y = -m_1(x)\left(m_2(x)\left(m_3 y'\right)'\right)' + [q(x)+ir(x)+\lambda]y = f(x), \quad f \in L_p,\ \lambda \ge 1$$

of the third order differential equations was considered in the dissertation work of R.D.Akhmetkaliyeva «Coercive estimate of the solution of the singular differential equation and its applications» [3].

Definition. A function $y(x) \in L_p(R)$ is called a solution of the differential equation in the following form

$$L_\lambda y := -m(x)\left(m(x)y'\right)'' + [q(x)+ir(x)+\lambda]y = f(x),$$

if there exists a sequence $\{y_n\}_{n=1}^{\infty}$ of three times continuously differentiable functions with compact support , and $\|y_n - y\|_p \to 0$, $\|L_\lambda y_n - f\|_p \to 0$, ($n \to \infty$) are fulfilled.

A symbol $C^{(k)}(R)$ is signified the set of all k times continuously differentiable functions $\varphi(x)$. $\sum_{j=0}^{k} \sup_{x\in R}\left|\varphi^{(j)}(x)\right| \prec \infty$ holds for a functions $\varphi(x)$. Let $W_\lambda(x) := \dfrac{|q(x)+\lambda+ir(x)|}{5+x^2}$.

Our main results in this work read:

Theorem. Suppose that $q(x)$ and $r(x)$ are continuous functions on R and satisfies the following conditions:

$$\frac{q(x)}{25+10x^2+x^4} \ge 1,\ r(x) \ge 1,$$

$$c^{-1} \le \frac{q(x)}{q(\eta)},\ \frac{r(x)}{r(\eta)} \le c,\ x,\eta \in R,\ |x-\eta| \le 1 \text{ for some } c \succ 0$$

$$\sup_{|x-\eta|\le 1} \frac{|W_\lambda(x) - W_\lambda(\eta)|}{|W_\lambda(x)|^\nu |x-\eta|^\mu} \prec +\infty,\ 0 \prec \nu \prec \frac{\mu}{3}+1,\ \mu \in (0,1],\ \lambda \ge 0.$$

Then there is a number $\lambda_0 \ge 0$, such that there exists a unique solution y for all $\lambda \ge \lambda_0$ of the equation (1) and for it the estimate

$$\left\| \sqrt{5+x^2}\left(\sqrt{5+x^2}\,y'\right)'' \right\|_p^p + \left\|(q(x)+ir(x)+\lambda)y\right\|_p^p \le c\left\|f(x)\right\|_p^p \tag{2}$$

holds.

For proving the obtained results firstly, we construct the function in the following form

$$M_0(x,\eta,\lambda) = \begin{cases} -\dfrac{1}{3(5+x^2)}\dfrac{e^{i(x-\eta)\xi_1}}{\xi_1^2}, & -\infty \prec \eta \prec x, \\ \dfrac{1}{3(5+x^2)}\displaystyle\sum_{j=2}^{3}\dfrac{e^{i(x-\eta)\xi_j}}{\xi_j^2}, & x \prec \eta \prec +\infty. \end{cases}$$

Here $\xi_s = \xi_s(x)$ $(s = 1,2,3)$ are the roots of the equation:

$$(5+x^2)\xi^3 - r(x) + i(q(x)+\lambda) = 0.$$

Let $d(\eta) \in C_0^\infty(-1,1)$ be a function in the following form

$$d(\eta) = \begin{cases} 1, & |\eta| \le \dfrac{1}{2}, \\ 0, & |\eta| \ge 1. \end{cases}$$

We denote

$$M_1(x,\eta,\lambda) = \left[\left(q(\eta)+ir(\eta) - \frac{5+\eta^2}{5+x^2}(q(x)+ir(x))\right)\right] M_0(x,\eta,\lambda)d(\eta-x),$$

$$M_2(x,\eta,\lambda) = -\left[2\left(\sqrt{5+\eta^2}\right)'\sqrt{5+\eta^2}\,d(\eta-x) + 3(5+\eta^2)d'(\eta-x)\right]\frac{\partial^2 M_0(x,\eta,\lambda)}{\partial\eta^2} -$$

$$-\left[\left(\sqrt{5+\eta^2}\right)''\sqrt{5+\eta^2}\,d(\eta-x) + 4\left(\sqrt{5+x^2}\right)'\sqrt{5+\eta^2}\,d'(\eta-x) + 3(5+x^2)d''(\eta-x)\right]\frac{\partial M_0(x,\eta,\lambda)}{\partial\eta} -$$

$$-\left[\left(\sqrt{5+\eta^2}\right)''\sqrt{5+\eta^2}\,d'(\eta-x) + 2\left(\sqrt{5+x^2}\right)'\sqrt{5+\eta^2}\,d''(\eta-x) + (5+x^2)d'''(\eta-x)\right]M_0(x,\eta,\lambda).$$

We represent the next integral operators:

$$(M_j(\lambda)f)(\eta) = \int_R M_j(x,\eta,\lambda)f(x)dx \quad (j = 1,2,3).$$

Lemma 1. Let $1 \prec p \prec +\infty$, $k(x,\eta)$ be a continuous function and

$$(Kv)(\eta) := \int_R k(x,\eta)v(x)dx.$$

Then

$$\|K\|_{L_p \to L_p} \le \sup_{\eta \in R} \int_R [|k(x,\eta)| + |k(\eta,x)|]dx .$$

Lemma 2. Let all conditions of the Theorem 1 hold. Then the operators $M_j(\lambda)$, $j = 1,2,3$ are continuous in the L_p and they satisfies the following estimate:

$$\|M_1(\lambda)\|_{L_p \to L_p} \le \frac{c_1}{b_\lambda^{\mu+3-3\nu}(\eta)}, \quad \mu \in (0,1], \quad 0 \prec \nu \prec \frac{\mu}{3} + 1 \tag{3}$$

$$\|M_2(\lambda)\|_{L_p \to L_p} \le \frac{c_2}{b_\lambda(\eta)}, \tag{4}$$

and, also

$$\|M_3(\lambda)\|_{L_p \to L_p} \le \frac{c_3}{(5+\eta^2)b_\lambda^3(\eta)},$$

where $b_\lambda(x) = \sqrt[3]{\dfrac{|r(x) - i(q(x)+\lambda)|}{5+x^2}}$.

Lemma 3. Let all conditions of Theorem hold, then it satisfies the next equality:

$$L_\lambda[M_3(\lambda)f](\eta) = f(\eta) + [M_1(\lambda)f](\eta) + [M_3(\lambda)f](\eta). \tag{5}$$

Suppose that all conditions of Theorem holds for $q(x)$, $r(x)$ and let $\frac{1}{p} + \frac{1}{p'} = 1$, where p' is conjugate number of p. The symbol $(L_\lambda)'$ means the operators, which operate in the $L_{p'}(R)$, which is described by the next equality:

$$(L_\lambda y, z) = (y, (L_\lambda)' z), \quad y \in D(L_\lambda), \qquad z \in D((L_\lambda)').$$

Apparently, from this it leads to:

$$(L_\lambda)' z \equiv \left(\sqrt{5+x^2}\left(\sqrt{5+x^2}\,z\right)''\right)' + (q(x) + \lambda - ir(x))z .$$

We examine the third-order differential equation in the following form

$$(L_\lambda)' z \equiv \left(\sqrt{5+x^2}\left(\sqrt{5+x^2}\,z\right)''\right)' + (q(x) + \lambda - ir(x))z = g(x), \tag{6}$$

here $q(x)$, $r(x)$ are continuous functions with a real value, $\lambda \ge 1$ and $g(x) \in L_{p'}(R)$.

Lemma 4. Let all conditions of the Theorem hold for the continuous functions $q(x)$, $r(x)$. Then there is a number $\lambda_1 \ge 0$, such that equation (6) has the solution for all $\lambda \ge \lambda_1$.

Proof of Theorem. Applying the estimates (3) and (4) from Lemma 1, we make a conclusion that there is a number $\lambda_0 \ge 0$ such that the inequality $\|M_1(\lambda)\|_{L_p \to L_p} + \|M_2(\lambda)\|_{L_p \to L_p} \le \frac{1}{2}$ fulfills for $\lambda \ge \lambda_0$. Because of this there exists a bounded inverse operator $G^{-1}(\lambda)$ in L_p of the operator, which is defined in the next form: $G(\lambda) := E + M_1(\lambda) + M_2(\lambda)$. Consequently, assuming an equation $h = [E + M_1(\lambda) + M_2(\lambda)]f$, taking into account an equality (5) from the Lemma 2, we receive that $L_\lambda[M_3(\lambda)G^{-1}(\lambda)h]\eta = h$. So, it ensues that equation (1) for any right-hand side f has the solution.

We make a conclusion that there is an right inverse of the operator $(L_\lambda)'$, which operates in the space $L_{p'}(R)$ during the $\lambda \ge \lambda_1$ by applying Lemma 3. A right inverse is defined on $L_{p'}(R)$. So, $\ker\left((L_\lambda)'\right)^* = \{0\}$, here $\left((L_\lambda)'\right)^*$ is conjugate operator of $(L_\lambda)'$. Hence, we get that $\ker L_\lambda = \{0\}$, $\lambda \ge \tilde{\lambda} = \max(\lambda_0, \lambda_1)$ due to $\left((L_\lambda)'\right)^*$ is an extension of the operator L_λ. So, L_λ is bounded invertible operator in the space $L_{p'}(R)$. Actually, we obtain that

$$(L_\lambda)^{-1} = M_3(\lambda)G^{-1}(\lambda), \qquad \lambda \ge \tilde{\lambda} = \max(\lambda_0, \lambda_1) \tag{7}$$

Suppose that the equation (1) has a solution, and solution is y. Here $\lambda \ge \tilde{\lambda} = \max(\lambda_0, \lambda_1)$. We should prove the estimate (2) by applying (7), Lemma 1 and all conditions of the Theorem. We obtain that

$$\left\|(q+\lambda+ir)(L_\lambda)^{-1}\right\|_{L_p \to L_p} = \left\|(q+\lambda+ir)M_3(\lambda)G^{-1}(\lambda)\right\|_{L_p \to L_p} \le c\sup_{\eta\in R}\int_{\eta-1}^{\eta+1} b_\lambda^3(\eta)b_\lambda^{-2}(x)\exp\left[-\sigma|x-\eta|b_\lambda(x)\right]dx \le$$

$$\le c\sup_{\eta\in R} b_\lambda(\eta)\int_{\eta-1}^{\eta+1}\exp\left[-\sigma|x-\eta|b_\lambda(x)\right]dx \prec \infty.$$

Due to this and (1) we make a conclusion that $\left\|\sqrt{5+x^2}\left(\sqrt{5+x^2}\,y'\right)''\right\|_p \le c\left(\|f\|_p + \|y\|_p\right)$. Eventually, by combining the last two estimates we get (2). The Theorem is completely proved.

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BOUNDEDNESS OF THE GENERALIZED FRACTIONAL – MAXIMAL OPERATOR IN GLOBAL MORREY TYPE SPACES

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For a measurable function $\phi:(0,\infty) \to (0,\infty)$ the generalized fractional maximal operator M_ϕ is defined by

$$(M_\phi f)(x) = \sup_{t>0}\phi(t)\int_{B(x,t)}|f(y)|dy,$$

where $B(x,t)$ is the ball in R^n of radius r centered at $x \in R^n$.