Solving output control problems using Lyapunov gradient-velocity vector function

M. A. Beisenbi, Zh. O. Basheyeva
Department of Systems Analysis and Control, L.N.Gumilyov Eurasian National University, Astana, Kazakhstan

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ABSTRACT

This paper describes a controller and observer parameter definition approach in one input-one output (closed-loop) control systems using Lyapunov gradient-velocity vector function. Construction of the vector function is based on the gradient nature of the control systems and the parity of the vector functions with the potential function from the theory of catastrophe. Investigation of the closed-loop control system's stability and solution of the problem of controller (determining the coefficient of magnitude matrix) and observer (calculation of the matrix elements of the observing equipment) synthesis is based on the direct methods of Lyapunov. The approach allows to select parameters based on the requested characteristics of the system.

Corresponding Author:

Zh. O. Basheyeva,
Department of Systems Analysis and Control,
Eurasian National University,
2 Satpayev St. Astana, 010000, Kazakhstan.
Email: zhuldyz.basheyeva@gmail.com

1. INTRODUCTION

In practice the state vector is available for manipulation less often than the plant output. This leads to the use of the state value in the control law instead of the state variables received by the observers [1-4]. This, in return, requires that the dynamic properties of the system change accordingly. We aim to observe how the replacement of the state variables by state values affects the properties of the system. In the modal control [1, 5] case characteristic polynomials are found through output.

The characteristic polynomial [6] of the closed-loop system with a controller that uses state values with an observer requires that the roots of the polynomial with a modal control be combined with the observer's own number [1, 3, 6]. This way synthesizing the modal controller with the observer becomes a challenging task.

The known iterative algorithms [1, 3] of separate own value control are based on the preliminary matrix triangularization or block-diagonalization. Notice that the control matrix is used as the nonsingular transition matrix in this canonical transformation, and it is defined by its own vectors in complicated unequivocal ways described here [6, 7].

The paper considers the control systems with one input and one output. To research the dynamic equalizer we use the Lyapunov gradient-velocity vector function [8-11]. Construction of the vector function is based on the gradient nature of the control systems and the parity of the vector functions with the potential function from the theory of catastrophe [12, 13]. Investigation of the closed-loop control system's stability and solution of the problem of controller (determining the coefficient of magnitude matrix) and observer (calculation of the matrix elements of the observing equipment) synthesis is based on the direct methods of

Lyapunov [14-16]. The approach offered in this paper can be considered as a way of determining the parameters of the controller and observer for a closed-loop with certain transitional characteristics.

2. RESEARCH

Assume the control system can be described by this set of equations [1-4]:

$$\dot{x}(t) = Ax(t) + Bu(t), y(t) = Cx(t), x(t_0) = x_0, \tag{1}$$

$$u(t) = -K\hat{x}(t), \tag{2}$$

$$\dot{x}(t) = (A - LC)\hat{x}(t) + Bu(t) + Ly(t), \hat{x}(t_0) = \hat{x}_0, \tag{3}$$

Modify the state equation (1)-(3). For this we will use the estimation error $\varepsilon(t) = x(t) - \hat{x}(t)$. Then we can write is as: $\hat{x}(t) = x(t) - \varepsilon(t)$, and Equations (1)-(3) will transform to:

$$\dot{x}(t) = Ax(t) + BKx(t) + BK\varepsilon(t), x(t_0) = x_0, \tag{4}$$

$$\dot{\varepsilon}(t) = A\varepsilon(t), -LC\varepsilon(t), \varepsilon(t_0) = \varepsilon_0 \tag{5}$$

For brevity consider the system with one input and one output, hence the system looks like:

$$A = \begin{Vmatrix} 0 & 1 & 0 & ... & 0 \\ 0 & 0 & 1 & ... & 0 \\ ... & ... & ... & ... & ... \\ 0 & 0 & 0 & ... & 0 \\ -a_n & -a_{n-1} & -a_{n-2} & ... & -a_1 \end{Vmatrix}, B = \begin{Vmatrix} 0 \\ 0 \\ ... \\ 0 \\ b_n \end{Vmatrix}, L = \begin{Vmatrix} 0 \\ 0 \\ ... \\ 0 \\ l_n \end{Vmatrix}$$

$$K = \|k_1, k_2, ..., k_n\|, C = \|c_1, c_2, ..., c_n\|$$

The set of (4), (5) will transform into:

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = x_3 \\ ... \\ \dot{x}_{n-1} = x_n \\ \dot{x}_n = -(a_n + b_n k_1)x_1 - (a_{n-1} + b_n k_2)x_2 - (a_{n-2} + b_n k_3)x_3 -, ..., -(a_1 + b_n k_n)x_n + \\ + b_n k_1 \varepsilon_1 + + b_n k_2 \varepsilon_2 + b_n k_3 \varepsilon_3, ..., + b_n k_n \varepsilon_n \\ \varepsilon_1 = \varepsilon_2 \\ \varepsilon_2 = \varepsilon_3 \\ ... \\ \varepsilon_{n-1} = \varepsilon_n \\ \dot{\varepsilon}_n = -(a_n + l_n c_1)\varepsilon_1 - (a_{n-1} + l_n c_2)\varepsilon_2 - (a_{n-2} + l_n c_3)\varepsilon_3 -, ..., -(a_1 + l_n c_n)\varepsilon_n \end{cases} \tag{6}$$

Notice that in the absence of the external impact, the process in the set (4), (5) must asymptotically approach the processes of a system with a controller, as if the closed-loop system according to a state vector, was affected by the impact of the convergent disturbance waves. These disturbances are caused by the $K\varepsilon(t)$ polynom in the Equation (5). The error must converge and the speed of convergence is defined during the synthesis of the observer. The main property of the set (4) and (5) lies in the asymptomatical stability. This way we found the requirement for the asymptotical stability of the system using the gradient-velocity method of the Lyapunov functions [8-11].

From (6) we find the components of the vector gradient for the Lyapunov vector function $V(x,\varepsilon)=(V_1(x,\varepsilon),V_2(x,\varepsilon),...,V_{2n}(x,\varepsilon))$:

$$\begin{cases} \frac{\partial V_1(x,\varepsilon)}{\partial x_2}=-x_2;.\frac{\partial V_2(x,\varepsilon)}{\partial x_3}=-x_3;.\frac{\partial V_{n-1}(x,\varepsilon)}{\partial x_n}=-x_n; \\ \frac{\partial V_n(x,\varepsilon)}{\partial x_1}=(a_n+b_nk_1)x_1,\frac{\partial V_n(x,\varepsilon)}{\partial x_2}=(a_{n-1}+b_nk_2)x_2,...,\frac{\partial V_n(x,\varepsilon)}{\partial x_n}=(a_1+b_nk_n)x_n, \\ \frac{\partial V_n(x,\varepsilon)}{\partial \varepsilon_1}=-b_nk_1\varepsilon_1,\frac{\partial V_n(x,\varepsilon)}{\partial \varepsilon_2}=-b_nk_2\varepsilon_2,...,\frac{\partial V_n(x,\varepsilon)}{\partial \varepsilon_n}=-b_nk_n\varepsilon_n \\ \frac{\partial V_{n+1}(x,\varepsilon)}{\partial \varepsilon_1}=-\varepsilon_2;\frac{\partial V_{n+2}(x,\varepsilon)}{\partial \varepsilon_2}=-\varepsilon_3;..., \\ .\frac{\partial V_{2n}(x,\varepsilon)}{\partial \varepsilon_1}=(a_n+l_nc_1)\varepsilon_1,\frac{\partial V_{2n}(x,\varepsilon)}{\partial \varepsilon_2}=(a_{n-1}+l_nc_2)\varepsilon_2,...,\frac{\partial V_{2n}(x,\varepsilon)}{\partial \varepsilon_n}=(a_1+l_nc_n)\varepsilon_n \end{cases} \quad (7)$$

From (6) we find the decomposition of the velocity vector to the coordinates $(x_1,...,x_n,\varepsilon_1,...,\varepsilon_n)$.

$$\begin{cases} \left(\frac{dx_1}{dt}\right)_{x_2}=x_2,\left(\frac{dx_2}{dt}\right)_{x_3}=x_3,...,\left(\frac{dx_n}{dt}\right)_{x_n}=x_n, \\ \left(\frac{dx_n}{dt}\right)_{x_1}=-(a_n+b_nk_1)x_1,\left(\frac{dx_n}{dt}\right)_{x_2}=-(a_{n-1}+b_nk_2)x_2,...,\left(\frac{dx_n}{dt}\right)_{x_n}=-(a_1+b_nk_n)x_n, \\ \left(\frac{dx_n}{dt}\right)_{\varepsilon_1}=b_nk_1\varepsilon_1,\left(\frac{dx_n}{dt}\right)_{\varepsilon_2}=b_nk_2\varepsilon_2,...,\left(\frac{dx_n}{dt}\right)_{\varepsilon_n}=b_nk_n\varepsilon_n, \\ \left(\frac{d\varepsilon_1}{dt}\right)_{\varepsilon_1}=\varepsilon_2;\left(\frac{d\varepsilon_2}{dt}\right)_{\varepsilon_3}=\varepsilon_3;\left(\frac{d\varepsilon_{n-1}}{dt}\right)_{\varepsilon_n}=\varepsilon_n; \\ ... \\ \left(\frac{d\varepsilon_n}{dt}\right)_{\varepsilon_1}=-(a_n+l_nc_1)\varepsilon_1,\left(\frac{d\varepsilon_n}{dt}\right)_{\varepsilon_2}=-(a_{n-1}+l_nc_2)\varepsilon_2,...,\left(\frac{d\varepsilon_n}{dt}\right)_{\varepsilon_n}=-(a_1+l_nc_n)\varepsilon_n, \end{cases} \quad (8)$$

To research the stability of the system (6) we use the basics of Lyapunov's direct method [14-16]. For the system to achieve the asymptotical equilibrium we need to secure the existence of a positive function $V(x,\varepsilon)$ so that its total derivative on the time axis along the state function (6) is a negative function. The total derivative from Lyapunov function with regard to the state Equation (6) is defined as a scalar product of the gradient (7) from Lyapunov and the velocity vector. (8):

$$\begin{aligned} \frac{dV(x,\varepsilon)}{dt} &= \sum_{i=1}^{n}\sum_{k=1}^{n}\frac{\partial V_i(x,\varepsilon)}{\partial x_k}\left(\frac{dx_i}{dt}\right)_{x_k}+\sum_{i=n+1}^{2n}\sum_{k=1}^{n}\frac{\partial V_i(x,\varepsilon)}{\partial \varepsilon_k}\left(\frac{d\varepsilon_i}{dt}\right)_{\varepsilon_k}= \\ &=-x_2^2-x_3^2,...,-x_n^2-(a_n+b_nk_1)^2x_1^2- \\ &-(a_{n-1}+b_nk_2)^2x_2^2-(a_{n-2}+b_nk_3)^2x_3^2,...,-(a_1+b_nk_n)^2x_n^2-b_n^2k_1^2\varepsilon_1^2-b_n^2k_2^2\varepsilon_2^2- \\ &-,...,-b_n^2k_n^2\varepsilon_n^2-\varepsilon_2^2-\varepsilon_3^2-(a_n+l_nc_n)^2\varepsilon_1^2-(a_{n-1}+l_nc_2)^2\varepsilon_2^2,...,-(a_1+l_nc_n)^2\varepsilon_n^2 \end{aligned} \quad (9)$$

From (9) we derive that the total time derivative of the vector function will be negative. Lyapunov function from (7) can be represented in the scalar view:

$$V(x,\varepsilon)=-\frac{1}{2}x_2^2-\frac{1}{2}x_3^2,...,-\frac{1}{2}x_n^2+\frac{1}{2}(a_n+b_nk_1)x_1^2+\frac{1}{2}(a_{n-1}+b_nk_2)x_2^2++\frac{1}{2}(a_{n-2}+b_nk_3)x_3^2+,...,+$$
$$+\frac{1}{2}(a_1+b_nk_n)x_n^2-\frac{1}{2}b_nk_1\varepsilon_1^2-\frac{1}{2}b_nk_2\varepsilon_2^2-\frac{1}{2}b_nk_3\varepsilon_3^2,...,-\frac{1}{2}b_nk_n\varepsilon_n^2-\frac{1}{2}\varepsilon_2^2-\frac{1}{2}\varepsilon_3^2-,...,-\frac{1}{2}\varepsilon_n^2+$$
$$+\frac{1}{2}(a_n+l_nc_1)\varepsilon_1^2+\frac{1}{2}(a_{n-1}+l_nc_2)\varepsilon_2^2+\frac{1}{2}(a_{n-2}+l_nc_3)\varepsilon_3^2+,...,\frac{1}{2}(a_1+l_nc_n)\varepsilon_n^2=\frac{1}{2}(a_n+b_nk_1)x_1^2+$$
$$+\frac{1}{2}(a_{n-1}+b_nk_2-1)x_2^2+\frac{1}{2}(a_{n-2}+b_nk_3-1)x_3^2+,...,+\frac{1}{2}(a_1+b_nk_n-1)x_n^2+\frac{1}{2}(a_n+l_nc_1-b_nk_1)\varepsilon_1^2+$$
$$+\frac{1}{2}(a_{n-1}+l_nc_2-b_nk_2-1)\varepsilon_2^2+\frac{1}{2}(a_{n-2}+l_nc_3-b_nk_3-1)\varepsilon_3^2+,...,+\frac{1}{2}(a_1+l_nc_n-b_nk_n-1)\varepsilon_n^2, \quad (10)$$

The condition for the positive certainty (10) i.e. existence of Lyapunov function will be defined:

$$\begin{cases} a_n+b_nk_1>0 \\ a_{n-1}+b_nk_2-1>0 \\ a_{n-2}+b_nk_3-1>0 \\ ... \\ a_1+b_nk_n-1>0 \end{cases} \quad (11)$$

$$\begin{cases} a_n+l_nc_1-b_nk_1>0 \\ a_{n-1}+l_nc_2-b_nk_2-1>0 \\ a_{n-2}+l_nc_3-b_nk_3-1>0 \\ ... \\ a_1+l_nc_n-b_nk_n-1>0 \end{cases} \quad (12)$$

The quality and the stability of the control system is dictated by the elements of the matrix of the closed- system. That is determining the target values of coefficients in a closed-loop system will prodivide smooth transitional processes in a system and result in higher quality control. The set of inequalities (11) and (12) serve as the necessary condition for the robust dynamic equalizer. The condition (11) allows for the stability in the state vector. Imagine a control system with a set of desired transition processes with one input and one output:

$$\begin{cases} \dot{x}_1=x_2 \\ \dot{x}_2=x_3 \\ ... \\ \dot{x}_{n-1}=x_n \\ \dot{x}_n=-d_nx_1-d_{n-1}x_2-d_{n-2}x_3-,...,-d_1x_n \end{cases} \quad (13)$$

Explore the system (13) with the given coefficients $d_i\,(\mathrm{i}=1,...,\mathrm{n})$, using the gradient-velocity function [8]. From (13) we find the components of the gradient-vector function $V(x)=(V_1(x),...,V_n(x))$

$$\begin{cases} \dfrac{\partial V_1(x)}{\partial x_2}=-x_2;.\dfrac{\partial V_2(x)}{\partial x_3}=-x_3;...,\dfrac{\partial V_{n-1}(x)}{\partial x_n}=-x_n; \\ \dfrac{\partial V_n(x)}{\partial x_1}=d_nx_1,\dfrac{\partial V_n(x)}{\partial x_2}=d_{n-1}x_2,\dfrac{\partial V_n(x)}{\partial x_3}=d_{n-2}x_3,...,\dfrac{\partial V_n(x)}{\partial x_n}=d_nx_n \end{cases} \quad (14)$$

From (13) we determine the decomposition of the velocity vector according to the coordinates:

$$\begin{cases} \left(\dfrac{dx_1}{dt}\right)_{x_2}=x_2;.\left(\dfrac{dx_2}{dt}\right)_{x_3}=x_3;...,\left(\dfrac{dx_{n-1}}{dt}\right)_{x_n}=x_n; \\ \left(\dfrac{dx_n}{dt}\right)_{x_1}=-d_nx_1,\left(\dfrac{dx_n}{dt}\right)_{x_2}=-d_{n-1}x_2,\left(\dfrac{dx_n}{dt}\right)_{x_3}=-d_{n-2}x_3,...,\left(\dfrac{dx_n}{dt}\right)_{x_n}=-d_1x_n \end{cases} \quad (15)$$

The total time derivative from the vector function is defined as a scalr product of the gradient vector (14) and the velocity vector (15):

$$\frac{dV(x)}{dt} = -x_2 - x_3 -;...., -x_n^2 - d_n^2 x_1^2 - d_{n-1}^2 x_2^2 - d_{n-2}^2 x_3^2 -,..., -d_1^2 x_n^2 \quad (16)$$

From (16) follows that the total time derivative of the vector-gradient function is negative. The vector function in the scalar view from (14) can be represented as:

$$V(x) = \frac{1}{2} d_n x_1^2 + \frac{1}{2}(d_{n-1} - 1)x_2^2 + \frac{1}{2}(d_{n-2} - 1)x_3^2 +,...,+ \frac{1}{2}(d_1 - 1)x_n^2 \quad (17)$$

Now the task is to define the controller coefficients (the elements of the matrix K) so that they have values d_i This way exploring the stability of the system with set coefficients d_i will result in:

$$d_n > 0, d_{n-1} - 1 > 0, d_{n-2} - 1 > 0, ..., d_1 - 1 > 0, \quad (18)$$

Equalizing the elements of the set of inequalities (11) and (18) we will receive:

$$\begin{cases} k_1 = \frac{1}{b_n}(d_n - \mathrm{a_n}) \\ k_2 = \frac{1}{b_n}(d_{n-1} - \mathrm{a_{n-1}}) \\ k_3 = \frac{1}{b_n}(d_{n-2} - \mathrm{a_{n-2}}) \\ ... \\ k_n = \frac{1}{b_n}(d_1 - \mathrm{a_1}) \end{cases} \quad (19)$$

$$\begin{cases} 2a_n + l_n c_1 - d_n > 0 \\ 2a_{n-1} + l_n \mathrm{c_2} - \mathrm{d_{n-1}} - 1 > 0 \\ 2a_{n-2} + l_n \mathrm{c_3} - \mathrm{d_{n-2}} - 1 > 0 \\ ... \\ 2a_1 + l_n c_n - d_1 - 1 > 0 \end{cases} \quad (20)$$

From the set of inequalities (20) we will receive:

$$l_{\mathrm{n}} = \max\left(\frac{\mathrm{d_n} - 2a_n}{c_1}, \max_i \frac{\mathrm{d_{n-i}} + 1 - 2a_{n-i}}{c_{i+1}} \right), i = 1,2,..., n-1, \quad (21)$$

This way the Equations (4) and (5) allow us to conclude that in the absence of the external impact, the process in the system asymptotically approaches those in the system with a controller which state vector was affected by convergent disturbances. The role of the disturbances is played by the compound $Bk\varepsilon(t)$ in the equation (4). The speed of convergence of the error $\varepsilon(t)$ can be determined during the synthesis of the observer from (21).

3. CONCLUSION

The known methods of synthesis of the systems with controllers that use the state value and the observer are based on the combination of the roots of the characteristic polynomial with a modal control with the observer's own number. This process requires substantial calculations and requires preliminary matrix block-diagonalization. This nonsingular matrix in the canonical transformation is defined by its own vectors in. Researching the closed-loop control system using the gradient-velocity method of Lyapunov function gave us the opportunity to develop an approach for controller and observer parametrization that provides us with a system of our desire without extraneous calculations.

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BIOGRAPHIES OF AUTHORS

M.A. Beisenbi. Dr. Sci. (Engineering), Professor. Graduate of the Kazakh Polytechnic Institute V.I. Lenin, specialty "Automation and telemechanics" (1967-1972 gg.). Graduated postgraduate studies at the Moscow Higher Technical School Bauman (1977 - 1980). In 1982 in the MHTS Bauman successfully defended the candidate dissertation "Solution of optimal control problems for objects with distributed parameters in automated control systems", specialty 05.13.02 - System theory, theory of automatic control and system analysis. In 1998, defended doctoral thesis "Models and methods of analysis and synthesis of ultimately stable systems", specialty 05.13.01 - Management in technical systems, the Institute of Informatics and Management Problems of the Ministry of Education and Science of the Republic of Kazakhstan. Since 2002. - Head of the Department of ENU L.N. Gumilyov. Since 2002. - Member of the Dissertation Council for the defense of a doctoral dissertation at the L.N. Gumilev ENU. Since 2005. - Member of the Dissertation Council for the protection of a doctoral dissertation at K.I.Satpaev KazNTU. Teaching experience - 39 years.

Zh.O. Basheyeva. PhD Doctoral student of the Department of Automated Control Systems, L.N.Gumilyov Eurasian National University, Astana.