РУХАНИ ЖАҢҒЫРУ

20 АСТАНА

ЕУРАЗИЯ ҰЛТТЫҚ УНИВЕРСИТЕТІ

Студенттер мен жас ғалымдардың
«ҒЫЛЫМ ЖӘНЕ БІЛІМ - 2018»
XIII Халықаралық ғылыми конференциясы

СБОРНИК МАТЕРИАЛОВ

XIII Международная научная конференция студентов и молодых ученых
«НАУКА И ОБРАЗОВАНИЕ - 2018»

The XIII International Scientific Conference for Students and Young Scientists
«SCIENCE AND EDUCATION - 2018»

12th April 2018, Astana

ҚАЗАҚСТАН РЕСПУБЛИКАСЫ БІЛІМ ЖӘНЕ ҒЫЛЫМ МИНИСТРЛІГІ
Л.Н. ГУМИЛЕВ АТЫНДАҒЫ ЕУРАЗИЯ ҰЛТТЫҚ УНИВЕРСИТЕТІ

Студенттер мен жас ғалымдардың
«Ғылым және білім - 2018»
атты XIII Халықаралық ғылыми конференциясының
БАЯНДАМАЛАР ЖИНАҒЫ

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PROCEEDINGS
of the XIII International Scientific Conference
for students and young scholars
«Science and education - 2018»

2018 жыл 12 сәуір

Астана

УДК 378

ББК 74.58
Ғ 96

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«Ғылым және білім – 2018» атты студенттер мен жас ғалымдардың XIII Халықаралық ғылыми конференциясы = XIII Международная научная конференция студентов и молодых ученых «Наука и образование - 2018» = The XIII International Scientific Conference for students and young scholars «Science and education - 2018». – Астана: http://www.enu.kz/ru/nauka/nauka-i-obrazovanie/, 2018. – 7513 стр. (қазақша, орысша, ағылшынша).

ISBN 978-9965-31-997-6

Жинаққа студенттердің, магистранттардың, докторанттардың және жас ғалымдардың жаратылыстану-техникалық және гуманитарлық ғылымдардың өзекті мәселелері бойынша баяндамалары енгізілген.

The proceedings are the papers of students, undergraduates, doctoral students and young researchers on topical issues of natural and technical sciences and humanities.

В сборник вошли доклады студентов, магистрантов, докторантов и молодых ученых по актуальным вопросам естественно-технических и гуманитарных наук.

УДК 378

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ISBN 978-9965-31-997-6

$$\int_a^b f(t)\Delta t := F(b) - F(a),$$

for all $a,b \in T$. A function $F : T \to R$ is called an *antiderivative* of $f : T \to R$ provided

$$F^{\Delta}(t) = f(t)$$

holds for all $t \in T$.

Denote $[a;\infty)_{\mathrm{T}} := \{t \in T : t \geq a\}$, where T is a particular time scale, which is unbounded above. For $\forall a \in R$ the set of rd-continuous functions $f : [a;\infty)_{\mathrm{T}} \to R$ will be denoted by $C_{rd}[a;\infty)_{\mathrm{T}}$.

We consider an integral inequality in the following form:

$$\left(\int_a^{\infty} u^q(x)\left(\int_a^x f(t)\Delta t\right)^q \Delta x\right)^{1/q} \leq C\left(\int_a^{\infty} f^p(x)v^p(x)\Delta x\right)^{1/p}, \quad \forall f \in C_{rd}[a;\infty)_{\mathrm{T}}, \qquad (1)$$

where the constant C does not depend on function f, and p; q are fixed parameters and u and v be nonnegative weight functions. If $T = R$, then we get that the well known classical weighted Hardy type inequality was studied by the books [2], [3], [4] and [5].

Our main result reads as follows.

Therome 1. *Let* $1 \leq p \leq q < \infty$ *and* $1/p + 1/p' = 1$. *Then the inequality (1) holds if and only* $B < \infty$ *satisfied, where*

$$B := \sup_{t \in [a;\infty)_{\mathrm{T}}} \left(\int_t^{\infty} u^q(x)\Delta x\right)^{1/q} \left(\int_a^t v^{-p'}(\tau)\Delta \tau\right)^{1/p'} < \infty,$$

Moreover, $B \approx C$, *where* C *is the best constant in (1).*

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УДК 517.51

TWO-WEIGHTED ESTIMATES FOR RIEMANN-LIOUVILLE INTEGRAL ON TIME SCALE

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Since the discovery of the classical Hardy inequalities (continuous or discrete) much work has been done, and many papers which deal with new proofs, various generalizations and extensions have appeared in the literature. We refer the reader to the books [1-2] and the references cited therein.

A time scale T is an arbitrary closed subset of the real numbers R. The cases when the time scale is equal to the reals or to the integers represent the classical theories of integral and of discrete inequalities.

In this paper, without loss of generality, we assume that $\sup T = \infty$, and define the time scale interval $[a,b]_T$ by $[a,b]_T := [a,b] \bigcap T$. For more details of time scale analysis, we refer the reader to the two books by Bohner and Peterson [3], [4] which summarize and organize much of the time scale calculus. The three most popular examples of calculus on time scales are differential calculus, difference calculus, and quantum calculus, i.e, when $T = R$, $T = N$ and $T = q^{N_0} = \{q^t : t \in N_0\}$, where $T = R$. The forward jump operator and the backward jump operator are defined by:

$$\sigma(t) := \inf\{s \in T; t < s\}, \qquad \rho(t) := \inf\{s \in T; s < t\}.$$

A point $t \in T$, is said to be left–dense if $\rho(t) = t$, is right–dense if $\sigma(t) = t$, is left–scattered if $\rho(t) < t$ and right–scattered if $\sigma(t) > t$. A function $f : T \to R$ is said to be right–dense continuous (rd–continuous) provided f is continuous at right–dense points and at left–dense points in T, left hand limits exist and are finite. The set of all such rd–continuous functions is denoted by $C_{rd}([a,b]_T)$.

Assume $f : T \to R$ is a function and let $t \in T$. Then we define $f^{\Delta}(t)$ to be the number (provided it exists) with the property that given any $\varepsilon > 0$, there is a neighborhood U of t (i.e., $U = (t - \delta; t + \delta)$ \ T for some $\delta > 0$) such that

$$|[f(\sigma(t)) - f(s)] - f^{\Delta}(t)[\sigma(t) - s]| < \varepsilon\,|\sigma(t) - s|.$$

We call $f^{\Delta}(t)$ the delta (or Hilger) derivative of f at t [5]. If $F^{\Delta}(t) = f(t)$, then the Cauchy (delta) integral of $f(t)$ is defined by $\int_a^b f(t)\Delta t = F(b) - F(a)$. It can be shown (see [3]) that if $f \in C_{rd}([a,b]_T)$, then the Cauchy integral $F(t) = \int_{t_0}^{t} f(t)\Delta t$ exists, $t_0 \in T$, and satisfies $F^{\Delta}(t) = f(t)$, $t \in T$.

The functions g_0 and h_0 are $g_0(t,s) = h_0(t,s) = 1$ for all $t,s \in T$, and, given g_k and h_k for $k \in N_0$, the functions g_{k+1} and h_{k+1} are

$$g_{k+1}(t,s) = \int_{t_0}^{t} g_k(\sigma(t),s)\Delta t \quad \text{for all } t,s \in T,$$

and

$$h_{k+1}(t,s) = \int_{t_0}^{t} h_k(t,s)\Delta t \qquad \text{for all } t,s \in T,$$

If we let $h_k^{\Delta}(t,s)$ denote for each fixed s the derivative of $h_k(t,s)$ with respect to t, then

$$h_k^{\Delta}(t,s) = h_{k-1}(t,s) \qquad \text{for all } t,s \in T,$$

Similarly

$$g_k^{\Delta}(t,s) = g_{k-1}(\sigma(t),s) \qquad \text{for all } t,s \in T.$$

Hence, we can define the Riemann-Liouville integral operator:

$$R_h f(x) = \int_a^x h_{k-1}(x,s) f(s)\Delta s\,, \quad R_g f(x) = \int_a^x g_{k-1}(\sigma(x),s) f(s)\Delta s\,,$$

for $k \in N$ and $f \in C_{rd}([a,b]_T)$.

We consider an integral inequality in the following form:

$$\left(\int_a^b u^q(x)\left(R_h f(x)\right)^q \Delta x\right)^{1/q} \le C\left(\int_a^b f^p(x) v^p(x)\Delta x\right)^{1/p}, \quad \forall f \in C_{rd}[a;b]_{\mathrm{T}}, \qquad (1)$$

$$\left(\int_a^b u^q(x)\left(R_g f(x)\right)^q \Delta x\right)^{1/q} \le C\left(\int_a^b f^p(x) v^p(x)\Delta x\right)^{1/p}, \quad \forall f \in C_{rd}[a;b]_{\mathrm{T}}, \qquad (2)$$

where the constant C does not depend on function f, and p; q are fixed parameters and u and v be nonnegative weight functions. If $T = R$, then we get that the well known classical weighted Hardy type inequality was studied by the papers [6] and [7].

Our main result reads as follows.

Therome 1. *Let* $1 \le p \le q < \infty$ *and* $1/p + 1/p' = 1$. *Then the inequality (1) holds if and only* $B := \max\{B_1,\ B_2\} < \infty$ *satisfied, where*

$$B := \sup_{t\in[a;\infty)_{\mathrm{T}}}\left(\int_t^\infty h_{k-1}^q(x,s) u^q(x)\Delta x\right)^{1/q}\left(\int_a^t v^{-p'}(\tau)\Delta\tau\right)^{1/p'},$$

$$B := \sup_{t\in[a;\infty)_{\mathrm{T}}}\left(\int_t^\infty u^q(x)\Delta x\right)^{1/q}\left(\int_a^t h_{k-1}^{p'}(x,s) v^{-p'}(\tau)\Delta\tau\right)^{1/p'}$$

Moreover, $B \approx C$, *where* C *is the best constant in (1).*

Therome 2. *Let* $1 \le p \le q < \infty$ *and* $1/p + 1/p' = 1$. *Then the inequality (2) holds if and only* $A := \max\{A_1,\ A_2\} < \infty$ *satisfied, where*

$$B := \sup_{t\in[a;\infty)_{\mathrm{T}}}\left(\int_t^\infty g_{k-1}^q(\sigma(x),s) u^q(x)\Delta x\right)^{1/q}\left(\int_a^t v^{-p'}(\tau)\Delta\tau\right)^{1/p'},$$

$$B := \sup_{t\in[a;\infty)_{\mathrm{T}}}\left(\int_t^\infty u^q(x)\Delta x\right)^{1/q}\left(\int_a^t g_{k-1}^{p'}(\sigma(x),s) v^{-p'}(\tau)\Delta\tau\right)^{1/p'}$$

Moreover, $B \approx C$, *where* C *is the best constant in (1).*

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УДК 517.51

WEIGHTED HARDY-TYPE INEQUALITY FOR FRACTIONAL SUM IN h-DISCRETE FRACTIONAL CALCULUS

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The theory of h-discrete fractional calculus is a developing areas recently drawing attention from both theoretical and applied disciplines. During the last decades, There have been of great interest on this calculus and have been studied by many authors, we refer the reader to (see [1], [2], [3], [4], [5], [6]) and references therein. Also study its applications in many fields of mathematics (see [7], [8], [9]). However, h-discrete fractional calculus represent a very new area for scientists. It is a subject of applied mathematics that has proved to be very useful in applied fields such as economics, engineering, and physics (see [10], [11], [12], [13], [14]).

Now, we state the some preliminary results of the h-discrete fractional calculus which will be used throughout this paper.

Let $h>0$ and $\mathbb{T}_a=\{a,a+h,a+2h,\cdots\}$ with $\forall a\in\mathbb{R}$.

Definition 1. *(see [15]) Let $f:\mathbb{T}_a\to\mathbb{R}$. Then the h-derivative of the function $f(x)$ is defined by*

$$D_h f(t):=\frac{f(\delta_h(t))-f(t)}{h},\quad t\in\mathbb{T}_a,$$

where $\delta_h(t)=t+h$.

Definition 2. *(see [15]) Let $f:\mathbb{T}_a\to\mathbb{R}$. Then the h-integral (h-difference sum) is given*

$$\int_a^b f(x)d_hx:=\sum_{k=a/h}^{b/h-1} f(kh)h=\sum_{k=0}^{\frac{b-a}{h}-1} f(a+kh)h,$$

for $a,b\in\mathbb{T}_a : b>a$.

Let $D_hF(x)=f(x)$. Then $F(x)$ is called an h-antiderivative of $f(x)$ and is denoted by $\int f(x)d_hx$ [11]. If $F(x)$ is an h-antiderivative of $f(x)$ and $b\in\mathbb{T}_a$, we have that

$$\int_a^b f(x)d_hx:=F(b)-F(a).$$

Definition 3. *(see [15]) Let $t,\alpha\in\mathbb{R}$. Then the h-fractional function is defined by*

$$t_h^{(\alpha)}:=h^\alpha\frac{\Gamma(\frac{t}{h}+1)}{\Gamma(\frac{t}{h}+1-\alpha)},$$