

The interaction of fine particles with an expanding gas flow under fluidization conditions is considered. The objects of study are finely dispersed materials, their single particles, gas flow in a fluidized layer. The study used the laws of dynamics and hydrodynamics, the classical laws of mechanics, as well as mathematical methods for the analytical solution of equations. It is emphasized that when a particle moves upwards in a gas jet, three forces act on it: the resistance force F_c , the gravity force P , and the Archimedes force A . As a result, the motion of a fine particle in an expanding gas flow is described taking into account the law of dynamics. During the study, an analytical equation was obtained to determine the velocity of a particle during its rise and fall in a gas jet. During the study, an analytical equation was obtained to find the height of the particle ascent depending on the gas flow rate for given geometric parameters of the gas flow. The obtained formulas can be used in the process of studying the process of convective drying of finely dispersed materials for various design parameters of the dryer. In practice, as a rule, there are various empirical formulas that describe such interactions of particles for specific parameters, which make it difficult to generalize them. In this work, the correctness of the assumed conditions necessary for the analytical solution of the differential equation of particle motion is proved. As a result, formulas were obtained that make it possible to determine the velocity of a particle in a gas jet and the height of its rise depending on the gas flow rate. On the basis of these formulas, graphic dependences of the gas velocity in the jet on the height $V_g=f(Z)$, as well as the dependences of the height of the particle rise h_m on the air flow rate in the jet L at different jet expansion angles $\alpha=15^\circ$; $\alpha=20^\circ$; $\alpha=30^\circ$ are plotted. It was found that with an increase in the height Z in an expanding jet, the gas velocity in the jet V_g decreases, with an increase in the air flow rate in the jet L , the height of the particle rise h_m increases. These formulas are the basis for further consideration of the movement of particles in a fluidized layer in the process of convective drying of fine materials for its intensification of the drying process

Keywords: fluidized layer, convective drying, gas flows, fine particles, dryer, gas jet, heat and mass transfer

OBTAINING A FORMULA DESCRIBING THE INTERACTION OF FINE PARTICLES WITH AN EXPANDING GAS FLOW IN A FLUID LAYER

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1. Introduction

Convective drying of finely dispersed materials is designed to remove moisture from the material by evaporating moisture and removing the resulting vapors together with the coolant [1]. As is known, mathematical models of drying, various differential and analytical equations are often used to describe convective drying [1]. Generally, drying is carried out to improve the quality of the processed material and increase its shelf life [2]. Dryers differ from each other in various ways. Heated air, gas or vapor mixtures can be used as a drying agent [3–5]. Under certain conditions, drying performs the role of sterilization, including with the use of a steam flow. For example, it was found that the total number

of microbes in 1 g of feed meal of animal origin is completely eliminated through the use of multi-stage sterilization technology [4]. Improving the quality of fodder meal of animal origin when using drying-grinding equipment is associated with a reduction in the duration of drying and grinding of raw materials due to the uniformity of high temperature fields and the distribution of humidity inside the device, which reduces energy costs [5] and improves the interaction of crushed particles with a hot air flow. In the technological line for the production of dry animal feed, considerable attention is paid to the drying process. In this case, the production of fine particles is preceded by coarse force grinding, as well as cooking, pressing, which creates favorable conditions for final drying and fine grinding in a vortex gas flow [6]. Intensive

dehydration is facilitated by convective drying with simultaneous grinding. The supply of hot air through a fan into the drying chamber, where a rotating rotor with impact elements operates, contributes to efficient heat and mass transfer, which has a positive effect on the production of dry fine particles of feed meal of animal origin [7], dehydrated by a spouting gas flow. In the modern production of dry fine particles, electric drives are an obligatory part, without which it is impossible to organize mass industrial production. Often, the operation of drying equipment is ensured by the synchronous rotation of several electric motors connected to each other mechanically, electrically, or technologically [8], which is reflected in the interaction of dried particles in the hot gas flow. Based on previous studies, when planning and carrying out drying, particular importance should be paid to the efficiency of contacting dried particles in a variable gas flow, in particular, under fluidized layer conditions, which significantly affects the rationality of drying and the design parameters of the drying unit.

The use of a fluidized layer (or sometimes they say a fluidized layer) in the drying process made it possible to significantly intensify the process of convective drying of dispersed materials that are relatively easily fluidized by the coolant. This significantly increases the phase contact and the relative velocity of the phases, it is important to achieve a uniform and stable fluidization of the dried material, the absence of channeling and clumping of the material, as well as the stabilization of particle fluidization [9].

Today, there are numerous formulas of an empirical character that are used to determine the critical velocities and entrainment rates of dried particles in the drying chamber. However, it should be noted that the formulas differ quite a lot depending on the chosen conditions for conducting experimental studies.

As is known, the process of convective drying mainly depends on hydrodynamics, i. e. on the velocity of fine particles in the dryer, their interaction with the drying agent.

In this regard, there is a need for a theoretical approach to studying the interaction of dried particles with a coolant (heated air). Therefore, a theoretical study of the interaction of particles in an air flow is relevant for the intensification of the process of convective drying of fine materials.

2. Literature review and problem statement

For most heat and mass transfer processes, the instability of the gas flow leads to a decrease in productivity. Using hybrid dynamics modeling and experiments with multiphase flow, two fundamental mechanisms underlying the flow instability in a dense gas and solid flow were elucidated in [10] in particular, these are nonlinear drag and dissipation upon collision. As a result, it has been clarified how particles fluidized by gas exhibit «particle-wave» duality. Based on this, it has been experimentally shown that dense granules fluidized with gas can be synchronized into «ordered» structures by creating an adaptively excitatory fluid wave. The introduction of an additional fluid wave makes it possible to fine-tune the flow structure. It has been demonstrated that the flow of a gas and a solid begins with the movement of particle convection caused by the non-linear drag of the gas. This instability is further exacerbated by dissipative collisions between particles. In general, attention is paid to the instability of the flow of a dense gas-solid flow. However, a detailed study of an individual single particle in a changing gas flow has not been

studied, which could contribute to a more in-depth consideration of dissipative particle collisions. Separate particles of dispersed materials have not been studied separately due to the formulation of problems for studying the expanded volume of particles, i.e. from large to ultra-small, which is a major scientific work that requires additional scientific research.

In [11], the hydrodynamics of a gas-solid flow in a multi-stage fluidized layer was experimentally studied. The results showed that the total pressure drop across the bed had a good linear relationship with the ratio of the reduced gas velocity and the solids circulation velocity. In the middle transport section of the multi-stage fluidized bed, a relatively uniform upward flow was observed across the cross section. Intense gas-solid interaction in the expanding part of the multistage fluidized layer has been confirmed. In addition, under limited operating conditions, a uniform upward flow of slurry was observed with minimal back-mixing of solids at the top of the expanding section. Finally, the structure of the gas-solid flow in the expanding part of the multi-stage fluidized bed was depicted. However, in this work, attention is paid only to the hydrodynamics of activated carbon. For a comparative characterization of the study of the hydrodynamics of a gas-solid flow in a multistage fluidized layer, no information is given for other solid fine particles, which narrows the practical applicability of the results of scientific research. At the same time, it should be assumed that the results of studies on a multi-stage fluidized layer to improve the efficiency of activated carbon desulfurization can be taken into account for other identical substances.

In [12], a new phase diagram based on the coordination number correlation was proposed. It can be used to identify different flow regimes, or distinguish between fixed, expanded, and fluidized (layered) states of a population of particles. The transition of flow regimes is actually associated with a change in the microstructure. Thus, the new phase diagram also highlights the relationship between the macroscopic and microscopic properties of fluid particle flows. Such insights can be useful in continuous modeling of fluid flow, particle flow, and individual particles, but how to do this is a challenging area for further study.

Analysis of the probability density distribution showed that the new fluidized layer can reduce the segregation of gas and solid particles, as well as enhance the interaction of gas and solid particles [13]. Higher solids content and more uniform distribution of solids existed in the diameter expansion region, especially for the rapidly turbulent fluidized layer with the vortex annular feeder structure. However, in this work, when considering the flow dynamics and contact efficiency in a new rapidly turbulent fluidized layer with internal elements of the ring feeder, close attention is not paid to the movement of single particles of fine materials. Apparently, this was not included in the research objectives. The study of the gas-solid single substance contact could contribute to the development of scientific research in the field of particle movement and the flow of particles in a gas stream.

In a gas-solid fluidized layer with the introduction of gas exclusively into the core or the annular zone separately, changes in the hydrodynamic behavior are observed and thus the distribution of the electrostatic potential is regulated and controlled. From the experiments indicated in [14], it can be seen that an electropositive potential is observed in the upper zone of particle circulation, while the lower part has electronegative characteristics. Compared to conventional gas inlet conditions, a fluidized layer with a special core-ring

gas inlet structure behaves differently. As the surface velocity of the gas increases, the positively charged zone as well as the upper circulation zone expands when the fluidizing gas only enters the main zone; on the contrary, the negatively charged zone expands at a given gas velocity when the fluidizing gas first enters the annular zone. At lower surface velocities, the absolute value of the electrostatic potential at the center is smaller than at the wall, and the two potential values approach each other as the velocity becomes higher. In addition, in order to understand the hydrodynamics and the electrostatic field, a preliminary discussion of the mechanism of the influence of the flow pattern on the distribution of the electrostatic potential was put forward. However, in this paper, insufficient attention is paid to determining the velocity of a single particle in a gas jet. Apparently, this is due to the lack of a task. The implementation of this calculation will help to determine the main parameters that affect the intensification of the technological process of drying fine particles in a changing gas flow.

Particle properties are important factors influencing gas fluidization. In [15], the influence of the size and shape of particles on the characteristics of fluidization is studied using a combined approach of computational fluid dynamics for the gas phase and the discrete element method for particles. It shows that the layer becomes loose when the particle size becomes small. The minimum fluidization rate increases exponentially with increasing particle size. Ellipsoidal particles are predicted to have higher minimum bubbling velocities compared to spherical particles. «Chain phenomenon» exists in expanded and fluidized layers for fine elongated particles. At the same time, it should be noted that insufficient attention is paid in this work to the interaction of a single particle with an expanding gas flow. A detailed study of particles of the most common geometric shape is important for obtaining analytical equations that affect the solution of hydrodynamic problems.

The micromechanics of various regimes of liquid and particle flow, such as fixed, expanded and fluidized layers, during gas fluidization has been studied. To establish a connection between macroscopic and microscopic descriptions of complex fluid and particle flows, the focus is on the following two aspects: the formation of a stable extended layer with respect to cohesive forces between particles, sliding and rolling friction forces, and correlations between coordination forces. It is shown that the adhesion force is of decisive importance for the formation of a static swollen layer, while the forces of sliding and rolling friction also play a role. The layer expansion criterion is analyzed on a volume scale and on a particle scale, and a disadvantage is revealed on a volume scale [16]. However, no detailed attention is paid to mathematical methods for the analytical solution of equations. It is for determining the main parameters of a single particle of fine materials depending on the gas flow rate, trajectory and dynamics of fine particles in a convective gas flow. Finding these parameters can serve to study the process of convective drying of fine materials with various designs of the dryer. The solution to this issue is complex and requires additional research.

The [17] analyzes the effect of polydispersity and shape of particles in size, as well as the orientation and strength of the external magnetic field on the rheology of gas-liquefied layers of small magnetizable particles. The yield strength, gas flow permeability and gas velocity at the transition of the jamming of stabilized magnetic fluidized layers were measured. However, the work does not pay attention to the interaction of fine particles with an expanding gas flow in a fluidized layer. Most likely this is due to the lack of a task.

Anti-jet mills with fluidized layer are capable of meeting the ever-increasing demand for non-polluting fine particles. In this type of jet mill, solid material is carried away and accelerated by expanding gas jets that are focused on a focal point within the fluidized layer. As a result of collision of particles cause breakage. The process is affected by the relative velocities of the particles and the number of particle collisions. It is clear that both quantities are distributed. However, neither the relative particle velocities nor the collision frequencies in such units have been determined so far. This paper presents an innovative method for the experimental evaluation of stress conditions in jet mills. For this, mixtures of glass and plastic metal microspheres were used, the latter being used in small quantities. Interparticle collisions between aluminum and glass spheres lead to the formation of dents on the microparticles. The size and number of these dents are related to the individual collision velocities and the overall collision frequency. The correlation between dent size and impact velocity was derived from finite element calculations based on empirical data. The proposed approach was validated by measuring particle image velocity during re-injection of gas into the fluidized layer reactor. In this case, the influence of the distance between two opposite nozzles was studied. For a laboratory of anti-jet mill with fluidized layer, the effect of gas pressure and delay was investigated. The relative particle velocities turned out to be significantly lower than the gas velocities, and the number of contacts per particle turned out to be extremely high [18]. However, in the work, insufficient attention is paid to determining the height of the particle ascent depending on the flow rate of gas jets and the given geometric parameters of the gas jet flow. Determination of this dependence will help to regulate the flow rate of gas jets. The solution to this issue is complex and requires additional research.

Fluidized layers of granular materials can be stabilized by attractive interparticle forces that give the expanded layer an elastic modulus that stabilizes it against flow disturbances. It can be seen that stabilization in the structure of strong contacts occurs naturally due to universal van der Waals forces for layers with a particle size close to 50 μm , although in a very reduced range of gas velocities above the minimum fluidization velocity. As shown in [19], a magnetic field can induce attractive forces between magnetized particles, thereby expanding the range of stable fluidization. When the magnetic field is tilted, the particle chains tilt according to the balance between the magnetic attraction force, the particles, and the vertical resistance force, which lowers the magnetic yield strength and therefore shortens the magnetic stabilization interval. However, in the work, insufficient attention is paid to determining the height of the particle ascent depending on the flow rate of gas jets and the given geometric parameters of the gas jet flow. Determination of this dependence will help to regulate the flow rate of gas jets. The solution to this issue is complex and requires additional research.

In order to understand the factors responsible for changes in the behavior of fine particles during fluidization at high temperatures, experimental work was carried out using a fluidized layer of heated gas with a size of 140×1000 mm [20]. Five fractions of the powder were tested in the temperature range from room temperature to 500 °C. The results show that the average size distribution significantly influences the fluidization behavior of the materials. In particular, significant differences were observed in the fluidization characteristics of the largest samples and the thinnest samples.

Minimum fluidization conditions were compared with the prediction of the Ergun equation. The comparison was satisfactory only when the experimental values of the layer porosity are taken into account. In fact, the non-monotonic trend of the minimum fluidization rate with increasing temperature cannot be explained only by the effect of temperature on fluid dynamics in the reservoir. But there are several more observable effects on fluidization behavior due to temperature increase that can be attributed to increased interparticle forces:

1) an increase in the peak pressure drop close to the fluidization minimum on the fluidization curve as the gas velocity increases;

2) an increase for the smallest samples of the hysteresis of the fluidization curves, taking into account the fluidization branches;

3) a large tendency of the formation to uniform expansion;

4) an increasing difference between the parameters of the Richardson-Zaki equation found using the fitting procedure in experiments and the parameters found using the Richardson-Zaki correlations and the theoretical final velocity. However, the work does not pay attention to the derivation and consideration of the differential equation for the interaction of a single particle with an expanding gas flow. The study of this problem is very complex, which requires additional scientific research.

The [21] shows the experimental results of a method for improving the fluidization of layers of nanoparticles. Three different field configurations were tested: parallel flow field, cross flow field, and variable field configurations. Nanoparticle agglomerates are naturally charged by contact and tribocharging mechanisms and are therefore agitated by an external field that enhances fluidization. Variable field configuration gives the best results. In this configuration, the electric field strength is higher in the lower part of the layer, while on the free surface it is practically negligible. Thus, the larger agglomerates, which tend to sink to the bottom of the layer due to segregation and usually interfere with uniform fluidization, are strongly agitated. It is believed that the strong mixing of larger agglomerates, which usually settle to the bottom of the bed, further homogenizes the distribution of the gas flow within the layer by destabilizing the development of gas channels near the gas distributor. On the other hand, smaller agglomerates near the free surface are weakly excited. Therefore, fluidization is greatly enhanced. However, as a result of the occurring phenomena, attention should also be paid to the contact of finely dispersed individual particles, their movement during ascent and descent in the jet of an expanding gas flow in a fluidized bed. In addition, strong and prolonged mixing contributes to undesirable segregation. The study of the movement of single particles with the derivation of analytical equations can help to obtain certain dependencies aimed at optimizing the changing flow of gas and particles in a fluidized bed. The solution to this issue is complex and requires additional research.

At the same time, it is worth noting the promise of fluidization in the interaction of fine particles with a changing gas flow. The ability to bring a system into its «ordered» state by tuning the interactions can effectively reduce internal energy dissipation, which can lead to a technological breakthrough.

Therefore, a detailed study and solution of such problems, the dynamics of the movement of fine particles in a convective gas flow, is required. Moreover, a detailed study of a single particle of finely dispersed materials in a changing gas flow, the study of the contact of a gas-solid single substance. There is also the determination of the velocity of a single particle in

a gas jet, the determination of the height of the rise of a particle depending on the flow rate of gas jets and the specified geometric parameters of the gas jet flow.

3. The aim and objectives of research

The aim of research is to derive and consider the differential equation for the interaction of a single particle with an expanding gas flow.

The objectives of research are:

– to consider the motion of a single particle in a gas flow based on the law of dynamics;

– to obtain an analytical equation for determining the velocity of a particle during its rise and the velocity of falling of a particle in a gas jet;

– to obtain an analytical equation for determining the height of the rise of the particle, depending on the gas flow rate and the given geometric parameters of the gas flow.

4. Materials and methods of research

The objects of research were finely dispersed materials, their single particles, gas flow in a fluidized layer.

First, it was necessary to obtain theoretical equations for describing the velocity and height of ascent for a single particle in a gas jet. Then, to generalize the equations obtained for the entire fluidized layer, taking into account the porosity and height of the fixed layer. Also, the angle of inclination of the jet relative to the horizontal axis, obtain a generalized equation for calculating the height of the rise of particles in a real fluidized layer of the processed dispersed material.

To consider the motion of a single particle in a gas flow, the laws of dynamics and hydrodynamics, general laws of mechanics, particularly, the laws of conservation of motion, mass and energy are used. Mathematical methods were used for the analytical solution of equations using calculations, comparative analysis. They were to obtain analytical equations in order to determine the velocity of a particle during its rise and falling in a gas jet. Also, it has assisted to determine the height of the rise of a particle depending on the gas flow rate and the given geometric parameters of the gas flow. In the study of a fluidized layer, phenomena other than physical ones, described by mathematical equations, were studied.

Literature sources on the formation of a fluidized layer of materials in a gas flow are mainly based on experimental and empirical data, which differ significantly from each other [22].

5. Research results of studying the interaction of fine particles with an expanding gas flow in a fluidized layer

5.1. Obtaining a differential equation of motion of a particle in a gas jet

Let's consider the motion of a single particle in a gas flow, it is based on the law of dynamics:

$$a_m \cdot m_m = \sum_{k=0}^n F_k, \quad (1)$$

where a_m – the particle acceleration vector, m/s²; m_m – the particle mass, kg; $\sum_{k=0}^n F_k$ – the vector sum of the forces acting on the particle, N.

Thus, let's proceed from the general equation of dynamics.

Let's consider the vertical motion of a single particle of a spherical shape with a diameter d and density ρ_m in an expanding jet of a gas flow through a rectangular opening of the grating with a width a and a length b (Fig. 1).

Let's proceed from the fact that the movement of dispersed materials in an expanding gas flow in a «fluidized» layer is based on the following basic provisions:

- at the lower and upper points of particle motion in the flow, its velocity is equal to zero (points where the direction of motion of the particle changes);

- in the trajectory of particles movement there are sections where the particles are accelerated in the direction of the trajectory, where the deceleration of the particles occurs;

- in the acceleration section, the directions of the total impact force and velocity coincide, and in the deceleration section, these directions are opposite.

Let's denote the gas expansion angle as α . The volume flow rate of gas through this jet will be denoted by L .

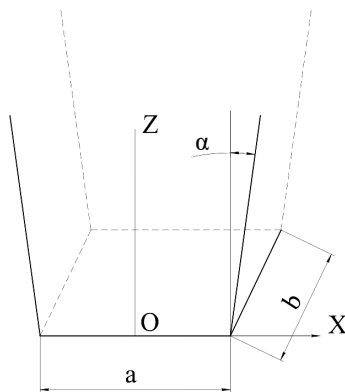


Fig. 1. Diagram of gas jet

Under these conditions and known values of the flow geometry a , b and α , the following relationship can be established between the gas velocity V_g in the jet and its volume flow:

$$V_g = \frac{L}{a \cdot b + 2 \cdot Z \cdot b \cdot \operatorname{tg} \alpha} = \frac{L}{b \cdot (a + 2 \cdot Z \cdot \operatorname{tg} \alpha)} \quad (2)$$

This equation describes the velocity of the gas in the jet depending on the height and its flow rate.

Let's build the dependence of the air velocity in the jet on the height Z with the following data: $a = 4 \cdot 10^{-3}$ m; $b = 0.1$; $L = 1.71 \cdot 10^{-2}$ m³/s.

In this case, the calculation is carried out at different angles of jet expansion: $\alpha = 15^\circ$; $\alpha = 20^\circ$; $\alpha = 30^\circ$.

The calculated data are listed in Table 1.

Estimated data

Table 1

| $\alpha = 15^\circ$ | | | | | | |
|---------------------|-------|-------|-------|-------|-------|-------|
| Z, m | 0.02 | 0.03 | 0.04 | 0.06 | 0.08 | 0.1 |
| V_g , m/s | 12.43 | 8.964 | 7.075 | 4.978 | 3.84 | 3.125 |
| $\alpha = 20^\circ$ | | | | | | |
| Z, m | 0.02 | 0.03 | 0.04 | 0.06 | 0.08 | 0.1 |
| V_g , m/s | 9.19 | 6.628 | 5.18 | 3.58 | 2.76 | 2.23 |
| $\alpha = 30^\circ$ | | | | | | |
| Z, m | 0.02 | 0.03 | 0.04 | 0.06 | 0.08 | 0.1 |
| V_g , m/s | 6.33 | 4.385 | 3.42 | 2.33 | 1.774 | 1.43 |

The graph of dependences $V_g = f(Z)$ at different angles of jet expansion is shown in Fig. 2.

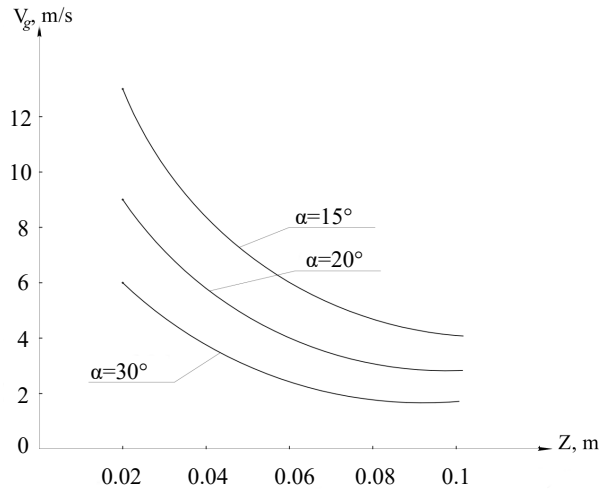


Fig. 2. Air velocity versus height in an expanding stream

Dynamic (1) can be written in terms of particle velocity:

$$m_m \cdot \frac{dV_m}{d\tau} = \sum F_i \quad (3)$$

The equation of dynamics is written in terms of the change in particle velocity.

When a particle moves upwards in a gas jet, three forces act on it: the resistance force F_c , the gravity force P and the Archimedes force A . Substituting the values of these forces into expression (3), let's obtain the following expanded equation:

$$V_m \cdot \frac{dV_m}{dZ} = \xi \cdot \frac{3 \cdot \rho_g}{4 \cdot d \cdot \rho_m} \cdot (V_g - V_m)^2 - g \cdot \left(\frac{\rho_m - \rho_g}{\rho_m} \right) \quad (4)$$

where ξ – coefficient of hydraulic resistance of the particle; ρ_g – gas density, kg/m³; ρ_m – particle density, kg/m³; respectively; V_g , V_m – velocity of gas and particles, m/s; g – free fall acceleration, m/s²; d – particle diameter, m.

(4) can be represented in the following form by introducing constant values of K and M :

$$V_m \cdot \frac{dV_m}{dZ} = K \cdot (V_g - V_m)^2 - M \quad (5)$$

where

$$K = \xi \cdot \frac{3 \cdot \rho_g}{4 \cdot d \cdot \rho_m}, \text{ m}^{-1};$$

$$M = g \cdot \left(\frac{\rho_m - \rho_g}{\rho_m} \right), \text{ m/s}^2.$$

Thus, an equation was obtained for the velocity of a particle in a gas jet depending on the physical parameters of the particle, the velocity of the gas. In the future, this differential equation will have to be solved analytically.

5. 2. Obtaining a formula for describing the velocity of a particle in a gas jet

(5) cannot be solved analytically. Therefore, for the purpose of an analytical solution, justifying, let's try to simplify it.

Analytically, this equation (5) is solved under one of the following assumptions:

1) over the entire integration section ($0 \leq Z \leq h_m$). It can be assumed with sufficient accuracy for practice that the velocity of a single material is many times less than the gas velocity $V_m \ll V_g$. The velocity of the particle is much smaller than the velocity of the gas. Here h_m is the maximum height of the particle in the gas jet, m. In this case, in the first term of the right side of (5), it is possible to neglect the particle velocity V_m and write $(V_g - V_m)^2 \approx V_g^2$;

2) the first term of the right side of (5) is many times smaller than the second term, i. e. $K \cdot (V_g - V_m)^2 \ll M$. In the deceleration zone of the particle on the right side of equation (5), the first term becomes much smaller than the second term.

When one of these two assumptions is satisfied, (5) is solved analytically.

Let's start with justifying the correctness of the first assumption. To this end, let's consider three characteristic points (Fig. 3) when the particle rises:

1. Let's pay attention to point 0 (Fig. 3). As mentioned above, at point 0 in accordance with the initial condition $V_m=0$, while the gas velocity has its maximum value $V_g=V_{g\max}$. Therefore, the first case is valid for this point, and (5) can be written as:

$$V_m \cdot \frac{dV_m}{dZ} = K \cdot V_g^2 - M. \tag{6}$$

This equation refers to the point $Z=0$ when the particle's velocity is zero.

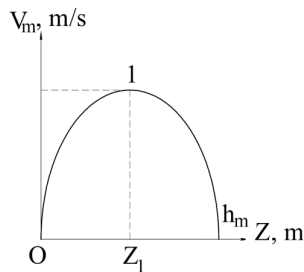


Fig. 3. Dependence of the particle velocity on the lift height

2. Let's pay attention to point 1 (Fig. 3). At point 1 the particle develops maximum velocity $V_m=V_{m\max}$, the gas velocity V_g has a slightly lower value than at point 0 and it is determined by (2). Therefore, at this point, the particle has a velocity that is as close as possible to the velocity of the gas. If even at this point the particle velocity at its maximum approach to V_g is incomparably small compared to the gas velocity ($V_m \ll V_g$), the particle velocity is much less than the gas velocity. Then this assumption will certainly be valid for other points of altitude.

Thus, it is necessary to consider point 1 (Fig. 3), i. e. transition point of the acceleration section to the deceleration section $Z=Z_1$.

For this point, the total force acting on the particle is zero, $\Delta R=0$:

$$\Delta R = F_c + A - P = 0. \tag{7}$$

Let's substitute the values of these forces into (7) and obtain:

$$V_g - V_m = \sqrt{\frac{4 \cdot d \cdot g \cdot (\rho_m - \rho_c)}{3 \cdot \xi \cdot \rho_m}}, \tag{8}$$

where V_g and V_m – velocities of the gas and the particle with respect to the coordinates adopted by us earlier, i. e. relative to the fixed grid.

According to formula (8), let's find the difference in the velocities of the gas and the particle at the point $Z=Z_1$. For example, with such given values of the physical and geometric parameters of the following specific values, $d=6 \cdot 10^{-3}$; $g=9.81$ m/s²; $\xi=0.44$; $\rho_m=960$ kg/m³; $\rho_c=1.29$ kg/m³ from equation (8) let's find:

$$V_g - V_m = \sqrt{\frac{4 \cdot 6 \cdot 10^{-3} \cdot 9.81 \cdot (960 - 1.29)}{3 \cdot 0.44 \cdot 1.29}} = 11.4 \text{ m/s}. \tag{9}$$

As mentioned above, the velocity of a particle at a point $Z=Z_1$ reaches its maximum value, so it is possible to substitute the value V_m instead of $V_{m\max}$.

Then the velocity difference:

$$V_{g1} - V_{m\max} = 11.4 \text{ m/s}. \tag{10}$$

Based on equation (10), let's construct the dependence of the ratio $(V_{g1} - V_{m\max}) \cdot 100\% / V_{g1}$ on the value $V_{m\max}$ (Table 2).

Table 2
Relationship $(V_{g1} - V_{m\max}) \cdot 100\% / V_{g1}$ versus value $V_{m\max}$

| $V_{m\max}$, m/s | 0 | 0.5 | 1.0 | 2.0 | 3.0 | 4.0 |
|---|-----|-------|-------|-------|------|------|
| $(V_{g1} - V_{m\max}) \cdot 100\% / V_{g1}$ | 100 | 95.80 | 91.94 | 85.07 | 79.2 | 74.0 |

This dependence is graphically shown in Fig. 4. From this dependence it can be seen that if the maximum particle velocity $V_{m\max}$ has a value not exceeding 1.5 m/s, then if the particle velocity in (6) is neglected, the error will not exceed 10%, which is acceptable error in solving hydrodynamic problems.

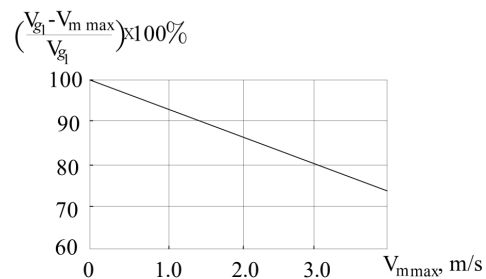


Fig. 4. Dependency graph of relationship from the value $V_{m\max}$

From this, it is possible to draw the following conclusion for the point $Z=Z_1$: in order to solve (6) without taking V_m into account on the right side of the equation and at the same time obtain a correct, from a practical point of view, result, it is necessary that the value of the maximum particle velocity be within $V_{m\max} < 1.5$ m/s. The maximum velocity of the particle is less than 1.5 m/s.

Let's solve (6). Then, from the solution, let's determine the value of $V_{m\max}$. If the calculated value is within these

limits, then assume that the formulation and solution of the problem are correct and vice versa.

3. At point 2 (Fig. 3), the velocity V_{g2} has a value slightly lower than at point $Z=0$ due to the expansion of the flow, determined by (2), and the particle velocity at this point V_{m2} is equal to zero. Therefore, (6) is also correct for this point.

Let's solve (6):

$$V_m \cdot \frac{dV_m}{dZ} = K \cdot V_g^2 - M,$$

or

$$V_m \cdot dV_m = K \cdot V_g^2 \cdot dZ - M \cdot dZ. \tag{11}$$

The gas velocity depends on the height Z according to (2). Substitute the value V_g from (2) into (11):

$$V_m \cdot dV_m = K \cdot \frac{L^2 \cdot dZ}{(a \cdot b + 2 \cdot b \cdot z \cdot \operatorname{tg} \alpha)^2} - M \cdot dZ. \tag{12}$$

Let's integrate both parts of the equation and get:

$$V_m^2 = \frac{K \cdot L^2}{a \cdot b^2 \cdot \operatorname{tg} \alpha} - \frac{K \cdot L^2}{a \cdot b^2 \cdot \operatorname{tg} \alpha + 2 \cdot b^2 \cdot Z \cdot \operatorname{tg}^2 \alpha} - 2 \cdot M \cdot Z. \tag{13}$$

The value of the volume flow L in (13) will be selected based on the condition of the required maximum height of the particle. Therefore, in order to estimate the ratio of velocities, first let's set, for example, the value $L=1.71 \cdot 10^{-2} \text{ m}^3/\text{s}$ and plot the dependence V_m on the height Z with the following data $a=4 \cdot 10^{-3} \text{ m}$; $b=0.1 \text{ m}$; $L=1.71 \cdot 10^{-2} \text{ m}^3/\text{s}$. The coefficients K and M will be chosen on the basis of formula (5) $K=0.075 \text{ m}^{-1}$; $M=9.796 \text{ m/s}^2$. Let's consider the braking zone ($Z_1 < Z \leq h_m$) separately. We have informed a few words about the stagnation zone from Z_1 to the point of maximum rise of the particle h_m (Fig. 3). Let's build a graph for various values of the jet expansion angle based on the calculated data from Table 3. Based on this table, let's plot the dependence $V_m=f(Z)$ (Fig. 5).

Comparing the data from Table 1, 3, it is possible to state that at all points in the braking zone, it is possible to read $V_m \ll V_g$. Let's assume that the particle velocity is much less than the gas velocity. The performed calculations show that in the zone of deceleration, the influence of gravity and drag forces increases. (5) can be replaced by (6) with sufficient accuracy for practical calculations, and the solution of (13) will be quite correct. Let's separately consider the section of particle acceleration during the rise of the particle due to the difference in the scale of the sections during acceleration and deceleration. Let's build the graph with the same data when considered the zone of inhibition.

Estimated data

| $\alpha=15^\circ$ | | | | | | |
|-------------------|-------|-------|-------|-------|-------|-------------|
| Z, m | 0.02 | 0.03 | 0.04 | 0.06 | 0.08 | 0.1 |
| V_m , m/s | 1.05 | 1.03 | 0.974 | 0.808 | 0.56 | ≈ 0 |
| $\alpha=20^\circ$ | | | | | | |
| Z, m | 0.02 | 0.03 | 0.04 | 0.05 | 0.06 | 0.07 |
| V_m , m/s | 0.892 | 0.83 | 0.738 | 0.616 | 0.455 | 0.167 |
| $\alpha=30^\circ$ | | | | | | |
| Z, m | 0.02 | 0.025 | 0.03 | 0.035 | 0.04 | 0.045 |
| V_m , m/s | 6.33 | 5.87 | 5.14 | 0.422 | 0.3 | 0 |

Table 3

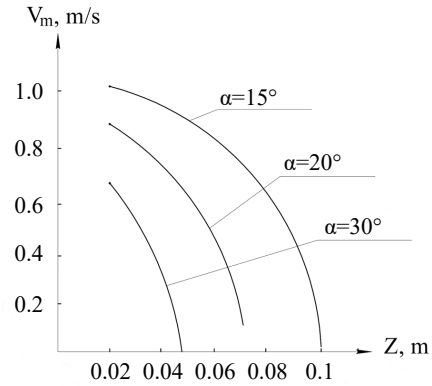


Fig. 5. Dependence of particle velocity on height in the area of its deceleration in an expanding jet

Let's enter the data in Table 4. Based on this table, let's plot the dependence $V_m=f(Z)$ (Fig. 6).

Table 4

Estimated data

| $\alpha=15^\circ$ | | | | | | |
|-------------------|---|-------|-------|-------|-------|-------|
| Z, m | 0 | 0.001 | 0.005 | 0.007 | 0.01 | 0.015 |
| V_m , m/s | 0 | 0.482 | 0.853 | 0.925 | 0.988 | 1.04 |
| $\alpha=20^\circ$ | | | | | | |
| Z, m | 0 | 0.001 | 0.005 | 0.007 | 0.01 | 0.015 |
| V_m , m/s | 0 | 0.472 | 0.774 | 0.833 | 0.886 | 0.899 |
| $\alpha=30^\circ$ | | | | | | |
| Z, m | 0 | 0.001 | 0.005 | 0.007 | 0.01 | 0.015 |
| V_m , m/s | 0 | 0.435 | 0.68 | 0.706 | 0.713 | 0.691 |

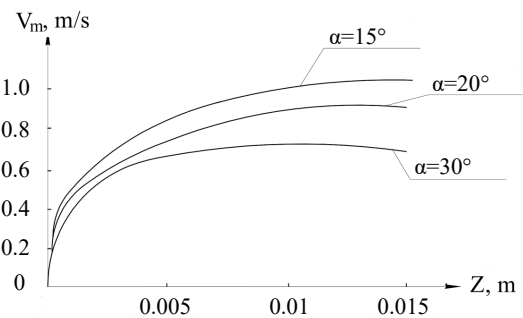


Fig. 6. Dependence of particle velocity on height in the area of its acceleration in an expanding jet

Based on the obtained graphical dependences of the particle velocity V_m on the height Z in the area of its acceleration in the expanding jet at different jet expansion angles $\alpha=15^\circ$; $\alpha=20^\circ$; $\alpha=30^\circ$, it was revealed that with an increase in the height Z in the section of its acceleration in the expanding jet, the particle velocity V_m increases. It is also seen that, as the jet expansion angle α decreases, the particle velocity V_m noticeably increases.

5.3. Obtaining a formula for determining the maximum height of a particle in a gas jet

From (13) one can find an expression for the maximum particle lift h_m . It is found from the condition that at $Z=h_m$ the particle velocity is $V_m=0$.

Considering these two conditions in (13), it is possible to find an equation for determining the height of the particle rise:

$$h_m = \frac{K \cdot L^2 - M \cdot a^2 \cdot b^2}{2 \cdot M \cdot a \cdot b^2 \cdot \text{tg}\alpha} \tag{14}$$

This formula makes it possible to calculate the height of the particle ascent for given values of the gas flow rate, the geometric parameters of the particle and the gas jet.

It is possible to calculate the height of the rise of the particle for given values of the air flow rate and the geometric dimensions of the jet and the particle.

For example, when $a=4 \cdot 10^{-3}$ m; $b=0.1$ m; $K=0.114$ m⁻¹; $M=9.793$ m/s².

The calculation is carried out at different jet expansion angles (Table 5).

Estimated data

Table 5

| α=15° | | | | | | |
|-------------------------------------|------|------|------|-------|-------|-------|
| $L \cdot 10^2, \text{m}^3/\text{s}$ | 0.5 | 1.0 | 2 | 3 | 4 | 5 |
| $h_m \cdot 10^2, \text{m}$ | 0.15 | 2.83 | 13.6 | 31.4 | 56.4 | 88.6 |
| α=20° | | | | | | |
| $L \cdot 10^2, \text{m}^3/\text{s}$ | 0.5 | 1 | 2 | 3 | 4 | 5 |
| $h_m \cdot 10^2, \text{m}$ | 1.1 | 2.08 | 9.98 | 23.16 | 41.58 | 65.27 |
| α=30° | | | | | | |
| $L \cdot 10^2, \text{m}^3/\text{s}$ | 1 | 1.5 | 2 | 3 | 4 | 5 |
| $h_m \cdot 10^2, \text{m}$ | 1.31 | 2.74 | 6.29 | 14.6 | 26.2 | 41.14 |

Based on these calculated data, let's construct the dependence of the lift height on the air flow (Fig. 7).

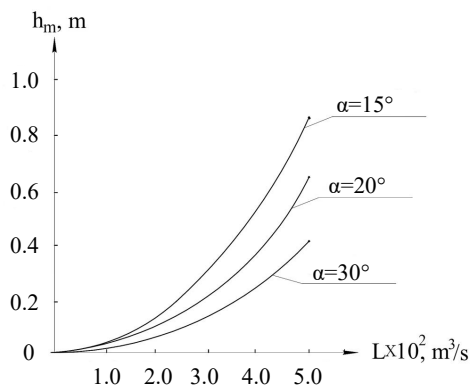


Fig. 7. The dependence of the height of the particle from the air flow in the jet

Based on the obtained graphical dependences of the particle lift height h_m on the air flow rate in the jet L at different jet expansion angles $\alpha=15^\circ$; $\alpha=20^\circ$; $\alpha=30^\circ$. It was revealed that with an increase in air flow rate in the jet L , the particle lift height h_m increases. It can also be seen that, as the jet expansion angle α decreases, the height h_m of the particle rise noticeably increases. The obtained numerical values and dependences determine the interaction of fine particles in a changing air flow during fluidization and can be useful in intensifying the drying process.

To check the correctness of the obtained formulas, let's present comparative calculations with known empirical for-

mulas obtained on the basis of experiments, which allow to determine the particle entrainment velocity (particle hover velocity). The most famous of the empirical equations is the following formula [23]:

$$\text{Re}_{cr} = \frac{Ar}{1,400 + 5.22\sqrt{Ar}} \tag{15}$$

It should be noted that the entrainment velocity of particles in the fluidized layer is equal to the critical velocity of a single particle.

From equation (14) for a single particle, one can find the condition for the particle to start moving:

$$K \cdot L^2 \geq M \cdot a^2 \cdot b^2 \text{ from here } L \geq ab \sqrt{\frac{M}{K}}$$

The entrainment velocity of a particle is determined from the gas velocity in the inlet section required for the particle to move:

$$V_{en} = V_{go} \geq \frac{L}{ab} = \sqrt{\frac{M}{K}} = 11.42 \frac{\text{m}}{\text{s}}$$

According to the above empirical Todes formula, the particle entrainment velocity is equal to $V_{en} = 11.04$ m/s.

Consequently, the discrepancy between the entrainment rates obtained on the basis of formulas (13), (14) obtained theoretically and the empirical Todes formula is about 3.4 %, which confirms their correctness.

Based on formula (14), taking into account the porosity ϵ_0 and height h_c of the fixed layer of dispersed material, a formula was obtained for the height of the rise of particles in a stable fluidized layer at a real gas flow L_d rate:

$$h_m = \frac{h_c \cdot (4 \cdot K \cdot L_d^2 - M \cdot a^2 \cdot b^2)}{2M \cdot a^2 \cdot b^2 - 4 \cdot K \cdot L_d^2} \tag{16}$$

For inclined gas jets (with an angle β relative to the horizontal axis), the following formula is obtained:

$$h_m = \frac{h_c \cdot \sin^2 \beta \cdot (4 \cdot K \cdot L_d^2 - M \cdot a^2 \cdot b^2)}{M \cdot a^2 \cdot b^2 (1 + \sin^2 \beta) - 4 \cdot K \cdot L_d^2 \sin^2 \beta} \tag{17}$$

In these formulas, the actual gas flow takes into account the porosity of the fixed layer of material, the interaction of neighboring jets. The equivalent diameter of the granular material is chosen as the particle diameter.

The critical velocity of particles in the fluidized layer V_{cr} , calculated by formula (17), gives the value $V_{cr}=1.074$ m/s. The same velocity, calculated by the empirical formula of Todes (15), is equal to 1.17 m/s, i. e. the difference does not exceed 8.2 %.

Thus, it is possible to say with confidence that the theoretically obtained formulas are well correlated with the known empirical dependencies. At the same time, the advantage of theoretical formulas is that they take into account the design features of the apparatus (the height of the fixed layer, the angle of inclination of gas jets, the gas flow rate in the jet, the geometry of the slots of the gas distribution grid) necessary for the engineering calculation of the apparatus.

6. Discussion of results about interaction of fine particles with an expanding gas flow in a fluidized layer

As is known, the hydrodynamic conditions inside the apparatus during the heat treatment of dispersed materials in a convective way determine the intensity of the process. In this case, hydrodynamic conditions are understood as the interaction of processed dispersed materials with the gas flow in the apparatus. Therefore, this paper considers hydrodynamic issues in apparatuses with a fluidized layer of dispersed materials.

In this work, the goal was set – to study not only the issues of fluidization of the processed material, but also its transportation inside the apparatus by a gas stream from inlet to outlet.

In practice, as the available literary sources show, there are many formulas of an empirical nature for determining the critical velocities of a substance in a fluidized bed, obtained on the basis of experimental data. In addition to the Todes formula (15) given above, there are a number of others that have been obtained empirically. In the literature [24] the empirical formula are proposed for determining the critical fluidization rate based on the Reynolds criterion.

The one formula based on the Lykov criterion is given in the work [25].

A number of other formulas obtained empirically could also be cited. They differ from each other in that they were obtained for different materials under certain experimental conditions. Due to the fact that these formulas were obtained for a particular case of experiments, attempts to generalize them present great difficulties. Another disadvantage of these formulas is that they do not consider the conditions of transportation of the processed material inside the apparatus.

In this regard, in this work, the emphasis was placed on the fact that to obtain a generalized formula for determining the critical velocity and height of the rise of particles in a fluidized layer theoretically based on the law of dynamics. For the fundamental nature of the approach, the interaction of a single particle with an expanding gas jet was initially considered on the basis of the law of dynamics. On its basis, the corresponding differential equation in the form (6) is obtained. The following is a study to determine the necessary conditions for the solvability of the resulting differential equation in an analytical way. The correctness of the assumed conditions for the analytical solution of the differential equation on the basis of relations (7)–(13) is proved.

Thus, taking into account the accepted conditions, the differential equation (6) was analytically solved and formula (14) was obtained for determining the rise height of a single particle in a gas jet.

After that, on the basis of this formula (14), for a single particle, taking into account the porosity ϵ_0 and the height h_c of the fixed layer of dispersed materials, formula (16) was obtained. It is to determine the height of the rise of a particle in a fluidized layer with a vertical supply of gas jets.

Taking into account the need for simultaneous transportation of the material in the apparatus by gas jets, formula (17) was obtained, where the angle β is the inclination of the gas jets relative to the horizontal axis.

The reliability of the obtained formulas was proved by comparing the calculated values with the well-known empirical Todes formula. The advantage of the formulas obtained by us is that they take into account the design features of the apparatus, the geometric and physical parameters of the

materials being processed. They are necessary for conducting the engineering calculation of the apparatus during heat treatment of dispersed materials in a fluidized layer with convective supply of a coolant (gas or air).

Due to the fact that these formulas were obtained for a particular case of experiments, attempts to generalize them present great difficulties.

In this regard, in this work, attention is paid to the theoretical consideration of the question of the interaction of a single particle with an expanding gas jet based on the law of dynamics. Theoretically, a differential equation is obtained for the velocity of particles in a gas flow, which, by and large, cannot be solved analytically.

To achieve the goal of an analytical solution of the obtained differential equation, some assumptions were made in this work. At the same time, evidence is provided for the validity of these assumptions.

On the basis of these justified approaches, formulas are obtained that describe the particle velocity and the maximum height of its ascent in a gas jet depending on the gas flow rate and the given physic-geometric parameters of the particle.

The obtained analytical formulas will be further applied in the study of the entire process of convective processing of finely dispersed materials in a fluidized layer, taking into account the design features of the apparatus.

The research results can be used for finely dispersed materials with known physical and geometric parameters.

7. Conclusions

1. The vertical motion of a single spherical particle with a diameter d and density ρ_m in an expanding jet of a gas flow through a rectangular opening of a lattice with a width a and a length b is considered. For known values of the flow geometry a , b and α , the relationship between the gas velocity V_g in the jet and its volumetric flow rate is established in the form:

$$V_g = \frac{L}{a \cdot b + 2 \cdot Z \cdot b \cdot \operatorname{tg} \alpha} = \frac{L}{b \cdot (a + 2 \cdot Z \cdot \operatorname{tg} \alpha)}$$

Graphical dependences of the gas velocity in the jet on the height $V_g = f(Z)$ are plotted for different angles of jet expansion $\alpha = 15^\circ$; $\alpha = 20^\circ$; $\alpha = 30^\circ$. It was found that with an increase in the height Z in an expanding jet, the gas velocity V_g in the jet decreases.

2. Based on the consideration of the differential equation for the interaction of a single particle with an expanding gas flow, analytical equations are obtained to determine the velocity of a particle during its rise and the velocity of a particle falling in a gas jet. The performed calculations show that in the zone of deceleration, the influence of gravity and drag forces increases. From the obtained dependence of the ratio $(V_{g1} - V_{m \max}) \cdot 100 \% / V_{g1}$ on the value $V_{m \max}$, it was revealed that if the maximum particle velocity has a value not exceeding 1,5 m/s, then if the particle velocity in the equation $V_m = dV_m/dZ = K \cdot V_g^2 - M$ is neglected, the error will not exceed 10 %, which is an acceptable error in solving hydrodynamic problems.

3. Based on the consideration of the differential equation for the interaction of a single particle with an expanding gas flow, an analytical equation was obtained to determine

the height of the particle rise h_m depending on the gas flow rate L and the given geometric parameters of the gas flow in the form:

$$h_m = \frac{K \cdot L^2 - M \cdot a^2 \cdot b^2}{2 \cdot M \cdot a \cdot b^2 \cdot \operatorname{tg} \alpha}.$$

Based on the obtained formula, taking into account the porosity and height of the fixed layer of dispersed material, generalized formulas were obtained for determining the height of the rise of particles. They will make it possible to carry out engineering calculations for a number of dryers. The dependences of the particle lift height h_m on the air flow

rate in the jet L at different jet expansion angles are plotted $\alpha=15^\circ$; $\alpha=20^\circ$; $\alpha=30^\circ$. It was found that with an increase in the air flow in the jet L , the height of the rise of the particle h_m increases. The obtained formulas can be the basis for further study of the process of convective drying of finely dispersed materials for various design parameters of the drying unit.

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