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DETERMINATION OF THE VALUES OF ARBITRARY CONSTANTS IN COSMOLOGICAL SOLUTIONS BASED ON OBSERVARIONAL DATA

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1. INTRODUCTION

In the past decade there is a general consensus that today our universe is undergoing an accelerated expansion. This result with the accelerated expansion becomes the central theme of the modern cosmology and confirms various observational evidences which are the observations of supernovae Type Ia (SNe Ia), cosmic microwave background radiation (CMB), and large-scale structure [1]. In this article, we compare the cosmological solution with observational data.

2. TELEPARALLEL GRAVITY WITH SCALAR FIELD

In order to consider the general action for the scalar field [1] that is not minimally coupled with the torsion scalar, we write:

$$S = \int d^4xe\left(F(\varphi)T + \frac{1}{2}\partial\mu\varphi\partial^\mu\varphi - V(\varphi)\right), \tag{1}$$

where $e = \det\left(e^i_\mu\right) = \sqrt{-g}$ that $e^i_\mu$is tetrad (vierbein) basis, T is a Torsion scalar, F(φ) is the generic function that describe the coupling and V (φ) is the potential for the scalar field.

The homogeneous and isotropic Friedmann-Robertson-Walker universe is described by the metric

$$ds^2 = dt^2 - a^2(t)(dx^2 + dy^2 + dz^2), \tag{2}$$

where a(t) is the scale factor. The torsion scalar in the teleparallel gravity can be expressed as $T = -\frac{6\dot{a}^2}{a^2}$. It is possible to obtain a point-like Lagrangian from action

$$L = -6F(\varphi)a^2\dot{a} + (\frac{1}{2}\dot{\varphi}^2 - V(\varphi)), \tag{3}$$

here, a dot indicates differentiation with respect to the cosmic time t. The Euler-Lagrange equations for a dynamical system are

$$\frac{\partial L}{\partial q_i} - \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q_i}}\right) = 0,$$
$$q = \{a, \varphi\}; \dot{q} = \{\dot{a}, \dot{\varphi}\}, \tag{4}$$

where q_i are the generalized coordinates of the configuration space. The configuration space of the Lagrangian (3) is $Q = (a, \varphi)$ and whose tangent space is $TQ = (a, \varphi, \dot{a}, \dot{\varphi})$. Substituting equation (3) into the Euler-Lagrange equation (4) for the scalar field, we obtain

$$\ddot{\varphi} + 3H\dot{\varphi} + 6F'^{H^2} + V' = 0, \tag{5}$$

with using the Hubble parameter, which can be expressed as a function of the scale factor $\frac{\dot{a}}{a} = H, \frac{\ddot{a}}{a} = H + \dot{H}^2$, which is a Klein Gordon equation for the coupled scalar field. From the Euler-Lagrange equation for the scale factor a by using the Lagrangian (3), we obtain the acceleration equation, namely

$$3H^2 + 2\dot{H} = -\frac{p_\varphi}{2F(\varphi)}. \tag{6}$$

$$3H^2 = \frac{2V(\varphi) - \dot{\varphi}^2}{4F(\varphi)}; \tag{7}$$

$$3H^2 = \rho. \tag{8}$$

3. NOETHER SYMMETRY AND COSMOLOGICAL SOLUTIONS

The Noether theorem generates a conserved quantity in the [1] classical mechanic. The application of this theorem to cosmology was introduced by De Rittis and Cappoziello to find preferred solutions of the field equations and the dynamical conserved quantity. The Noether theorem states that if the Lie derivative of a given Lagrangian L dragging along a vector field X vanishes

$$\mathcal{L}_X L = 0, \tag{9}$$

then X is a symmetry for the dynamics, and it generates the conserved quantity. Also the Noether symmetry approach allows one to choose the potential dynamically in the scalar-tensor gravity theory, and the explicit form of the function $f(R)$ of the modified $f(R)$ theories of gravity. In this form of $f(R)$, cosmological solutions in the case of FRW metric can describe the accelerated period of the Universe. The existence of Noether symmetry implies the existence of a vector field X such that

$$X = \alpha \frac{\partial L}{\partial a} + \beta \frac{\partial L}{\partial \varphi} + \dot{\alpha} \frac{\partial L}{\partial \dot{a}} + \dot{\beta} \frac{\partial L}{\partial \dot{\varphi}}, \tag{10}$$

where α, β and γ are depend on a and φ. Hence the condition given by (9) for the existence of a symmetry gives rise to the following set of coupled differential equations,

$$\alpha - 2\frac{\partial \alpha}{\partial a} a - \beta \frac{F'}{F} a = 0; \tag{11}$$

$$3\alpha + 2a \frac{\partial \beta}{\partial \varphi} = 0; \tag{12}$$

$$12F \frac{\partial \alpha}{\partial \varphi} - a^2 \frac{\partial \beta}{\partial a} = 0; \tag{13}$$

$$3V\alpha + aV\beta = 0. \tag{14}$$

This system are obtained by imposing the fact that the coefficients of $a^2, a\dot{\varphi}, \dot{\varphi}^2$vanish. Using the separation of variable one can find the solutions of the above set of differential equations (11)-(14) for α, β, coupling function $F(\varphi)$and potential $U(\varphi)$ as

$$F(\varphi) = \frac{(2n+3)^2}{48}\varphi^2, \tag{15}$$

$$V(\varphi) = \lambda\varphi^{\frac{6}{2n+3}}, \tag{16}$$

$$\beta = \alpha_0 a^s e^{\frac{3\ln\varphi}{2s+3}} = \alpha_0 a^n \varphi^{\frac{3}{2n+3}}, \tag{17}$$

$$\alpha = -a\beta\frac{V'}{3V} = -\frac{2\alpha_0}{2n+3}a^{n+1}\varphi^{-\frac{2n}{2n+3}}. \tag{18}$$

Next, we move on to new variables and get the scale factor

$$a(t) = \left(\frac{4\alpha_0 b_2}{3n+3}\right)^{-\frac{1}{2n}}(b_1 t + c_1)^{-\frac{3}{n(n+3)}}, \tag{19}$$

$$\varphi(t) = \left(\frac{4\alpha_0 b_2}{2n+3}\right)^{\frac{2n+3}{4n}}(b_1 t + c_1)^{\frac{2n+3}{2n}}. \tag{20}$$

Based on the equation (19), we found the Hubble parameter

$$H = \frac{\dot{a}}{a} = -\frac{3b_1}{n(3+n)(c_1+b_1 t)}.\ [2] \tag{21}$$

$$E = \frac{H(z)}{H(0)} = \frac{1}{(1+z)^{\frac{n^2+3n}{3}}}.\ [2] \tag{22}$$

The theoretical distance modulus is defined as

$$\mu_{th}(z_i) = 5\log_{10} D_L(z_i) + \mu_0 = 5\log_{10}\left(\frac{3(z+1)\left((z+1)^{1+n+\frac{n^2}{3}}-1\right)}{n^2+3n+3}\right) + 43{,}21746,\ [2] \tag{23}$$

where $\mu_0 = 42{,}38 - 5\log_{10} 0{,}68 = 43{,}21746$, whereas

$$D_L(z) = (1+z)\int_0^z \frac{1}{E}dz = (1+z)\int_0^z (1+z)^{\frac{n^2+3n}{3}}dz = \frac{3(z+1)\left((z+1)^{1+n+\frac{n^2}{3}}-1\right)}{n^2+3n+3}.\ [2] \tag{24}$$

4. OBSERVATION CONSTRAINTS

4.1. Comparison of observational data using the NonlinearModelFit

$$\text{NonlinearModelFit}[\text{data1}, 43.21746 + \frac{5\text{Log}[\frac{3(1+z)(-1+(1+z)^{1+n+\frac{n^2}{3}})}{3+3n+n^2}]}{\text{Log}[10]}, \{n\}, z];$$
$$\{n \to -1.0174839586461413\}$$

	"Estimate"	"Standard Error"	"t- Statistic"	"P- Value"
n	-1.0174839586461413	0.0856374005539506	-11.881303636781196	$2.754627562645906 \times 10^{-29}$

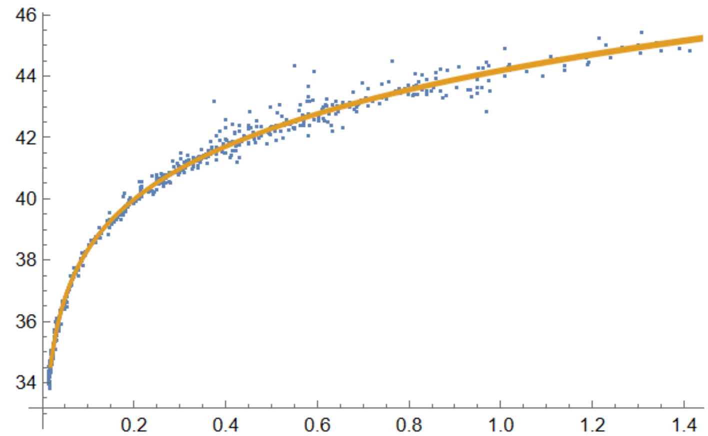
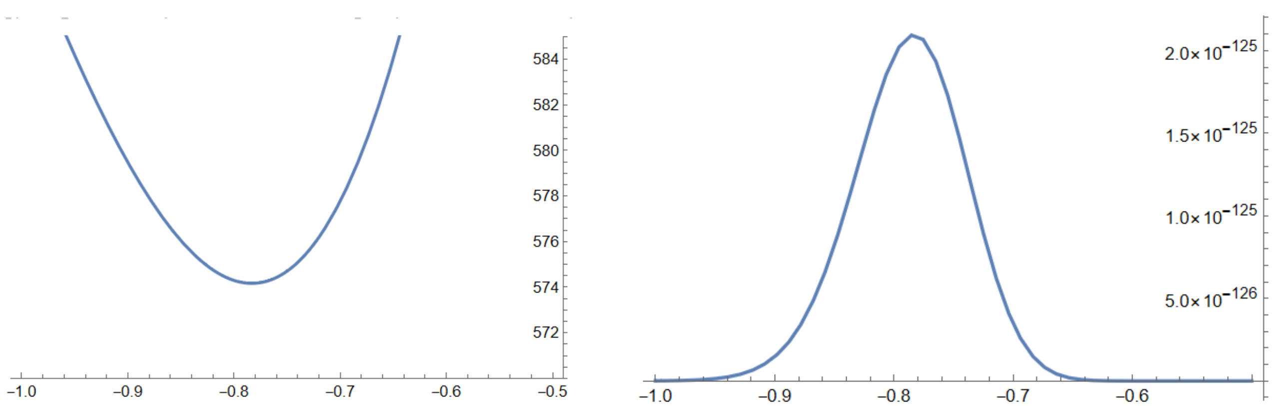


Fig. 1: $\{z, -0.5, -1\}$

4.2. Comparison of observational data using Markov Chain Monte Carlo (MCMC)

The Covariance Matrix	C_{ij}=(0.0020730963125789106)
The bestfit params	$\{-0.783190911696977 \pm 0.045531267416786356\}$
The mean params	$\{-0.7877833100662509 \pm 0.045540359236565804\}$



$\{n, -0.5, -1\}$ $\{n, -0.5, -1\}$

Fig. 2: The χ^2 and likelihood $\mathcal{L}_{\chi^2} \propto e^{-\frac{\chi^2}{2}}$ as functions of parameter n [2]

In this paper, we investigated a model of gravity with torsion and a non-minimally coupled scalar field. We applied Noether's theorem to find a cosmological solution. We found generators and then determined the conservation law for this system of equations. From the conservation law, we found a scale factor and estimated it using observational data. We used two methods for the assessment: 1) the "NonlinearModelFit" function from the Wolfram Mathematica package, and 2) the Monte Carlo method for Markov chains. The Monte Carlo method optimizes the free parameters of the model better than the built-in Wolfram Mathematica function.

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ПЕНЛЕВЕНІҢ ЕКІНШІ ТЕҢДЕУІН КОСМОЛОГИЯДА ҚОЛДАНУ

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Пенлеве теңдеуі космологияда маңызды рөл атқарады, гравитациялық толқындардың өзгерістерін зерттеуге және ғарыштық құрылымдарды қалыптастыруға қуатты математикалық құрал беріп, әлемнің іргелі динамикасын түсінуге ықпал етеді.

Ары қарай біртекті, изотропты және тегіс кеңістіктік Фридман-Робертсон-Уокер метрикасымен (ФРУ) бірге зерттеледі [1-5]

$$ds^2 = -dt^2 + a^2(t)\left(dx^2 + dy^2 + dz^2\right), \tag{1}$$

мұндағы: $a(t)$ - масштабты фактор.

Энергияның сақталу заңы

$$3H^2 = \rho, \tag{2}$$

$$\dot{\rho} = 6H\dot{H}, \tag{3}$$

$$3H^2 + 2\dot{H} = -p, \tag{4}$$

Сақталу заңы