UDC 517.957,532.5

STUDY OF COSMOLOGY AGAINST THE BACKGROUND OF THE GENERALIZED F(R, Q, X, φ) MODEL OF GRAVITY

Amangeldiyev Alikhan Nurkasymovich
aamangeldiev525@gmail.ru
4th-grade student of the ENU named after L.N. Gumilyov
Nur-Sultan, Kazakhstan
Scientific director: Yerzhanov K.K

At present, General Relativity (GR) is considered the best accepted fundamental theory describing gravity. GR is described in terms of the Levi-Civita connection, which is the basis of Riemannian geometry with the Ricci curvature scalar R. But GR can be described in terms of different geometries from the Riemannian one, for example, F(R) gravity. There are several other alternative gravity theories. For example, one of the alternative gravity theories is the so-called teleparallel gravity with the nonmetricity scalar Q or its generalization F(Q) gravity. Another possible alternative gravity theory is F(X, φ). In this paper, we will consider the more general gravity theory.

We have Lagrangian in the next form [1]:

$$L = a^3\left[F-(R-u)F_R-(Q-\omega)F_Q\right]-6a\dot{a}^2\left[F_R-F_Q\right]-$$
$$-6a^2\dot{a}\left[\dot{R}F_{RR}+\dot{Q}F_{RQ}+\dot{X}F_{RX}+\dot{\varphi}F_{R\varphi}\right]-a^3F_X\left[X-\frac{1}{2}\dot{\varphi}^2\right]. \qquad (1)$$

Here: R – curvature scalar, Q – nonmetricity scalar, X – kinetic term of the scalar field, φ – scalar field.

$$R = 6\left(\frac{\ddot{a}}{a}+\frac{\dot{a}^2}{a^2}\right)+u, \qquad (2)$$

$$Q = 6\frac{\dot{a}^2}{a^2}+\omega, \qquad (3)$$

$$X = \frac{1}{2}\dot{\varphi}^2. \qquad (4)$$

The noether symmetries approach:

We can write the Noether symmetry condition in the following form for the Lagrangian [2]:

$$XL = 0, \qquad (5)$$

here:

$$X = \alpha\frac{\partial}{\partial a}+\beta\frac{\partial}{\partial R}+\gamma\frac{\partial}{\partial Q}+\delta\frac{\partial}{\partial X}+\varepsilon\frac{\partial}{\partial \varphi}+\dot{\alpha}\frac{\partial}{\partial \dot{a}}+\dot{\beta}\frac{\partial}{\partial \dot{R}}+\dot{\gamma}\frac{\partial}{\partial \dot{Q}}+\dot{\delta}\frac{\partial}{\partial \dot{X}}+\dot{\varepsilon}\frac{\partial}{\partial \dot{\varphi}}. \qquad (6)$$

The functions α, β, γ, δ, ε depend of the variables a, R, Q, X, φ and then:

$$\dot{\alpha} = \alpha_a\dot{a}+\alpha_R\dot{R}+\alpha_Q\dot{Q}+\alpha_X\dot{X}+\alpha_\varphi\dot{\varphi}, \qquad (7)$$

$$\dot{\beta} = \beta_a\dot{a}+\beta_R\dot{R}+\beta_Q\dot{Q}+\beta_X\dot{X}+\beta_\varphi\dot{\varphi}, \qquad (8)$$

$$\dot{\gamma} = \gamma_a\dot{a}+\gamma_R\dot{R}+\gamma_Q\dot{Q}+\gamma_X\dot{X}+\gamma_\varphi\dot{\varphi}, \qquad (9)$$

$$\dot{\delta} = \delta_a \dot{a} + \delta_R \dot{R} + \delta_Q \dot{Q} + \delta_X \dot{X} + \delta_\varphi \dot{\varphi}, \quad (10)$$

$$\dot{\varepsilon} = \varepsilon_a \dot{a} + \varepsilon_R \dot{R} + \varepsilon_Q \dot{Q} + \varepsilon_X \dot{X} + \varepsilon_\varphi \dot{\varphi}. \quad (11)$$

By this, we have: For $F(R, Q, X, \varphi)$

$$\begin{aligned}
&\alpha 3a^2\left[F-(R-u)F_R-(Q-\omega)F_Q\right]+\alpha a^3\left[u_a F_R+\omega_a F_Q\right]-\alpha 6\dot{a}^2\left[F_R-F_Q\right]-\\
&-\alpha 12a\dot{a}\left[\dot{R}F_{RR}+\dot{Q}F_{RQ}+\dot{X}F_{RX}+\dot{\varphi}F_{R\varphi}\right]-\alpha 3a^2 F_X\left[X-\frac{1}{2}\dot{\varphi}^2\right]+\\
&+\beta a^3\left[-(R-u)F_{RR}-(Q-\omega)F_{QR}\right]-\beta 6a\dot{a}^2\left[F_{RR}-F_{QR}\right]-\\
&-\beta 6a^2\dot{a}\left[\dot{R}F_{RRR}+\dot{Q}F_{RQR}+\dot{X}F_{RXR}+\dot{\varphi}F_{R\varphi R}\right]-\beta a^3 F_{XR}\left[X-\frac{1}{2}\dot{\varphi}^2\right]+\\
&+\gamma a^3\left[-(R-u)F_{RQ}-(Q-\omega)F_{QQ}\right]-\gamma 6a\dot{a}^2\left[F_{RQ}-F_{QQ}\right]-\\
&-\gamma 6a^2\dot{a}\left[\dot{R}F_{RRQ}+\dot{Q}F_{RQQ}+\dot{X}F_{RXQ}+\dot{\varphi}F_{R\varphi Q}\right]-\gamma a^3 F_{XQ}\left[X-\frac{1}{2}\dot{\varphi}^2\right]+\\
&+\delta a^3\left[-(R-u)F_{RX}-(Q-\omega)F_{QX}\right]-\delta 6a\dot{a}^2\left[F_{RX}-F_{QX}\right]-\\
&-\delta 6a^2\dot{a}\left[\dot{R}F_{RRX}+\dot{Q}F_{RQX}+\dot{X}F_{RXX}+\dot{\varphi}F_{R\varphi X}\right]-\delta a^3 F_{XX}\left[X-\frac{1}{2}\dot{\varphi}^2\right]+\\
&+\varepsilon a^3\left[F_\varphi-(R-u)F_{R\varphi}-(Q-\omega)F_{Q\varphi}\right]-\varepsilon 6a\dot{a}^2\left[F_{R\varphi}-F_{Q\varphi}\right]-\\
&-\varepsilon 6a^2\dot{a}\left[\dot{R}F_{RR\varphi}+\dot{Q}F_{RQ\varphi}+\dot{X}F_{RX\varphi}+\dot{\varphi}F_{R\varphi\varphi}\right]-\varepsilon a^3 F_{X\varphi}\left[X-\frac{1}{2}\dot{\varphi}^2\right]-\\
&-\alpha_a 12a\dot{a}^2\left[F_R-F_Q\right]-\alpha_R\dot{R}12a\dot{a}\left[F_R-F_Q\right]-\alpha_Q\dot{Q}12a\dot{a}\left[F_R-F_Q\right]-\alpha_X\dot{X}12a\dot{a}\left[F_R-F_Q\right]-\\
&-\alpha_\varphi\dot{\varphi}12a\dot{a}\left[F_R-F_Q\right]+\alpha_a\dot{a}a^3[u_{\dot{a}}F_R+\omega_{\dot{a}}F_Q]+\alpha_R\dot{R}a^3[u_{\dot{a}}F_R+\omega_{\dot{a}}F_Q]+\\
&+\alpha_Q\dot{Q}a^3[u_{\dot{a}}F_R+\omega_{\dot{a}}F_Q]+\alpha_X\dot{X}a^3[u_{\dot{a}}F_R+\omega_{\dot{a}}F_Q]+\alpha_\varphi\dot{\varphi}a^3[u_{\dot{a}}F_R+\omega_{\dot{a}}F_Q]-\\
&-\alpha_a\dot{a}6a^2\left[\dot{R}F_{RR}+\dot{Q}F_{RQ}+\dot{X}F_{RX}+\dot{\varphi}F_{R\varphi}\right]-\alpha_R\dot{R}6a^2\left[\dot{R}F_{RR}+\dot{Q}F_{RQ}+\dot{X}F_{RX}+\dot{\varphi}F_{R\varphi}\right]-\\
&-\alpha_Q\dot{Q}6a^2\left[\dot{R}F_{RR}+\dot{Q}F_{RQ}+\dot{X}F_{RX}+\dot{\varphi}F_{R\varphi}\right]-\alpha_X\dot{X}6a^2\left[\dot{R}F_{RR}+\dot{Q}F_{RQ}+\dot{X}F_{RX}+\dot{\varphi}F_{R\varphi}\right]-\\
&-\alpha_\varphi\dot{\varphi}6a^2\left[\dot{R}F_{RR}+\dot{Q}F_{RQ}+\dot{X}F_{RX}+\dot{\varphi}F_{R\varphi}\right]-\beta_a 6a^2\dot{a}^2F_{RR}-\beta_R\dot{R}6a^2\dot{a}F_{RR}-\\
&-\beta_Q\dot{Q}6a^2\dot{a}F_{RR}-\beta_X\dot{X}6a^2\dot{a}F_{RR}-\beta_\varphi\dot{\varphi}6a^2\dot{a}F_{RR}-\delta_a 6a^2\dot{a}^2F_{RX}-\delta_R\dot{R}6a^2\dot{a}F_{RX}-\\
&-\delta_Q\dot{Q}6a^2\dot{a}F_{RX}+\varepsilon_R\dot{R}a^3F_X\dot{\varphi}+\varepsilon_Q\dot{Q}a^3F_X\dot{\varphi}+\varepsilon_X\dot{X}a^3F_X\dot{\varphi}+\varepsilon_\varphi a^3F_X\dot{\varphi}^2=0
\end{aligned} \quad (12)$$

From the resulting equation (12) we create a system of equations:

$$\begin{aligned}
&\dot{a}^2:-6\alpha\left[F_R-F_Q\right]-6\beta a\left[F_{RR}-F_{RQ}\right]-6\gamma a\left[F_{RQ}-F_{QQ}\right]-6\delta a\left[F_{RX}-F_{QX}\right]-6\varepsilon a\left[F_{R\varphi}-F_{Q\varphi}\right]-\\
&-12\alpha_a a\left[F_R-F_Q\right]-6\beta_a a^2F_{RR}-6\gamma_a a^2F_{RQ}-6\delta_a a^2F_{RX}-6\varepsilon_a a^2F_{R\varphi}=0,
\end{aligned} \quad (13)$$

$$\dot{R}^2: 6\alpha_R a^2 F_{RR} = 0, \quad (14)$$

$$\dot{Q}^2: 6\alpha_Q a^2 F_{RQ} = 0, \quad (15)$$

$$\dot{X}^2: 6\alpha_X a^2 F_{RX} = 0, \quad (16)$$

$$\dot{\varphi}^2 : \varepsilon_\varphi a^3 F_X - 6\alpha_\varphi a^2 F_{R\varphi} + \alpha \frac{3}{2} a^2 F_X + \beta \frac{1}{2} a^3 F_{XR} + \gamma \frac{1}{2} a^3 F_{XQ} + + \delta \frac{1}{2} a^3 F_{XX} + \varepsilon \frac{1}{2} a^3 F_{X\varphi} = 0, \tag{17}$$

$$\dot{a}\dot{R} : -12\alpha a F_{RR} - 6\beta a^2 F_{RRR} - 6\gamma a^2 F_{RRQ} - 6\delta a^2 F_{RRX} - 6\varepsilon a^2 F_{RR\varphi} - 12\alpha_R a\left[F_R - F_Q\right] - - 6\alpha_a a^2 F_{RR} - 6\beta_R a^2 F_{RR} - 6\gamma_R a^2 F_{RQ} - 6\delta_R a^2 F_{RX} - 6\varepsilon_R a^2 F_{R\varphi} = 0, \tag{18}$$

$$\dot{a}\dot{Q} : -12\alpha a F_{RQ} - 6\beta a^2 F_{RQR} - 6\gamma a^2 F_{RQQ} - 6\delta a^2 F_{RQX} - 6\varepsilon a^2 F_{RQ\varphi} - 12\alpha_Q a\left[F_R - F_Q\right] - - 6\alpha_a a^2 F_{RQ} - 6\beta_Q a^2 F_{RR} - 6\gamma_Q a^2 F_{RQ} - 6\delta_Q a^2 F_{RX} - 6\varepsilon_Q a^2 F_{R\varphi} = 0, \tag{19}$$

$$\dot{a}\dot{X} : -12\alpha a F_{RX} - 6\beta a^2 F_{RXR} - 6\gamma a^2 F_{RXQ} - 6\delta a^2 F_{RXX} - 6\varepsilon a^2 F_{RX\varphi} - 12\alpha_X a\left[F_R - F_Q\right] - - 6\alpha_a a^2 F_{RX} - 6\beta_X a^2 F_{RR} - 6\gamma_X a^2 F_{RQ} - 6\delta_X a^2 F_{RX} - 6\varepsilon_X a^2 F_{R\varphi} = 0, \tag{20}$$

$$\dot{a}\dot{\varphi} : -12\alpha a F_{R\varphi} - 6\beta a^2 F_{R\varphi R} - 6\gamma a^2 F_{R\varphi Q} - 6\delta a^2 F_{R\varphi X} - 6\varepsilon a^2 F_{R\varphi\varphi} - 12\alpha_\varphi a\left[F_R - F_Q\right] - - 6\alpha_a a^2 F_{R\varphi} - 6\beta_\varphi a^2 F_{RR} - 6\gamma_\varphi a^2 F_{RQ} - 6\delta_\varphi a^2 F_{RX} - 6\varepsilon_\varphi a^2 F_{R\varphi} - \varepsilon_a a^3 F_X = 0, \tag{21}$$

$$\dot{R}\dot{Q} : -6\alpha_R a^2 F_{RQ} - 6\alpha_Q a^2 F_{RR} = 0, \tag{22}$$

$$\dot{R}\dot{X} : -6\alpha_X a^2 F_{RR} - 6\alpha_R a^2 F_{RX} = 0, \tag{23}$$

$$\dot{Q}\dot{X} : -6\alpha_X a^2 F_{RQ} - 6\alpha_Q a^2 F_{RX} = 0, \tag{24}$$

$$\dot{R}\dot{\varphi} : -6\alpha_\varphi a^2 F_{RR} + \varepsilon_R a^3 F_X - 6\alpha_R a^2 F_{R\varphi} = 0, \tag{25}$$

$$\dot{Q}\dot{\varphi} : -6\alpha_\varphi a^2 F_{RQ} + \varepsilon_Q a^3 F_X - 6\alpha_Q a^2 F_{R\varphi} = 0, \tag{26}$$

$$\dot{X}\dot{\varphi} : -6\alpha_\varphi a^2 F_{RX} + \varepsilon_X a^3 F_X - 6\alpha_X a^2 F_{R\varphi} = 0. \tag{27}$$

$$\begin{aligned} &\alpha 3a^2\left[F - (R-u)F_R - (Q-\omega)F_Q + \frac{1}{3}a(u_a F_R + \omega_a F_Q)\right] + \\ &+ \beta a\left[-(R-u)F_{RR} - (Q-\omega)F_{QR}\right] + \gamma a\left[-(R-u)F_{RQ} - (Q-\omega)F_{QQ}\right] + \\ &+ \delta a\left[-(R-u)F_{RX} - (Q-\omega)F_{QX}\right] + \varepsilon a\left[F_\varphi - (R-u)F_{R\varphi} - (Q-\omega)F_{Q\varphi}\right] + \\ &+ \dot{a}\alpha_a a\left[u_{\dot{a}} F_R + \omega_{\dot{a}} F_Q\right] + \dot{R}\alpha_R a\left[u_{\dot{a}} F_R + \omega_{\dot{a}} F_Q\right] + \dot{Q}\alpha_Q a\left[u_{\dot{a}} F_R + \omega_{\dot{a}} F_Q\right] + \\ &+ \dot{X}\alpha_X a\left[u_{\dot{a}} F_R + \omega_{\dot{a}} F_Q\right] + \dot{\varphi}\alpha_\varphi a\left[u_{\dot{a}} F_R + \omega_{\dot{a}} F_Q\right] - (\alpha 3F_X + \beta a F_{XR} + \\ &+ \gamma a F_{XQ} + \delta a F_{XX} + \varepsilon a F_{X\varphi})X = 0. \end{aligned} \tag{28}$$

And we can combine equations (25), (26), (27). This will give us the next equations:

$$\dot{R}\dot{\varphi} : \varepsilon_R a^3 F_X = 6\alpha_\varphi a^2 F_{RR}, \tag{29}$$

$$\dot{Q}\dot{\varphi}: \varepsilon_Q a^3 F_X = 6\alpha_\varphi a^2 F_{RQ}, \tag{30}$$

$$\dot{X}\dot{\varphi}: \varepsilon_X a^3 F_X = 6\alpha_\varphi a^2 F_{RX}. \tag{31}$$

If we combine these equations we get the Monge – Ampere equation.

$$F_{RR}F_{XX} = F_{RX}^2 \tag{32}$$

The noether symmetries solution

From equations for (14), (15), and (16) we have two possibilities. First, $F_{RR} = F_{RQ} = F_{RX} = 0$ and solution to this is a linear equation

$$F = s_1(\varphi)R + s_2(\varphi)Q + s_3(\varphi)X + s_4(\varphi). \tag{23}$$

We can solve Monge – Ampere equation. This equation is homogeneous. After some calculations their solution involving arbitrary constants we can write as:

$$F = (C_1(\varphi)R + C_2(\varphi)Q + C_3(\varphi)X)^2 + C_4(\varphi)R + C_5(\varphi)Q + C_6(\varphi)X + C_7(\varphi). \tag{34}$$

This solution gives us the same results as recent observations about the early time inflation, close R^2. Solutions of Monge – Ampere equations involving one arbitrary function will give a more general result [3]:

$$F = f(C_1(\varphi)R + C_2(\varphi)Q + C_3(\varphi)X, \varphi) + C_4(\varphi)R + C_5(\varphi)Q + C_6(\varphi)X + C_7(\varphi). \tag{35}$$

Here we will use the only solution involving arbitrary constants with constants.

In this paper, it was possible to show that when considering a generalized model with a scalar field, including F(R) and F(Q) - gravity. If used the Noether symmetry method can get Starobinsky's solution. It is also important that Starobinsky's solution is obtained even if we consider separately only F(R) or only F(Q) - gravity.

References

1. K. Yerzhanov, G. Bauyrzhan, A. Altaibayeva, R. Myrzakulov. Inflation from the Symmetry of the Generalized Cosmological Model // Autumn – 2021 p. 5 – 9.

2. K. Yesmakhanova, N. Myrzakulov, S. Myrzakul, G. Yergaliyeva, K. Myrzakulov, K. Yerzhanov, R. Myrzakulov. Generalized gravity theory with curvature, tor,sion, and nonmetricity // 22 April 2021 p. 3 – 4.

3. R. Myrzakulov, K. Yerzhanov, G. Bauyrzhan, B. Meirbekov. F(R, T, X, φ) cosmology via Noether symmetry // 23 april 2021 p. 2 – 5.