UDK 524.832

CONSERVED CURRENTS IN MODIFIED THEORY OF GRAVITY

Amangeldi Alina Batyrzhankyzy
amangeldiab@mail.ru
L.N.Gumilyov ENU student of the Department of General and Theoretical Physics,
Nur-Sultan, Kazakhstan
Scientific adviser – Tsyba. P.Y

Modified theories of gravity or alternative theories of gravity are theories that generalize Einstein's theory of gravity. The purpose of alternative theories is to describe gravity within the framework of a modified theory, while offering a better description of phenomena in cosmology, and also not to contradict the available experimental data at the moment.

In fact, in many existing approaches to cosmological issues in modified gravity, the equations are reduced to second-order differential equations. There are a number of popular models of modified gravity: $f(R)$, Gauss-Bonnet theory $f(G)$ string theory, nonlocal gravity, Horav-Lifshitz $f(R)$ gravity and renormalized covariant gravity.

One of the most widely used are $f(R)$ type gravity and Gauss-Bonnet gravity $f(G)$. Gauss-Bonnet gravity $f(G)$ refers to scalar theories of gravity. It is assumed that this theory can describe the late epoch of cosmic acceleration [1].

$f(G)$ gravity, otherwise called Gauss-Bonnet gravity, is a modified version of an action that includes a Gauss-Bonnet invariant or a function depending on a given parameter.

The Gauss-Bonnet invariant is calculated by the formula

$$G = R^2 - 4R_{\mu\nu}R^{\mu\nu} + R_{\mu\nu\lambda\sigma}R^{\mu\nu\lambda\sigma}. \tag{1}$$

This model is considered within the framework of the Friedman-Robertson-Walker metric in 4-dimensions with the signature (−, +, +, +). The natural system of units of measurement is used $8\pi G = \hbar = c = 1$.

$$ds^2 = -dt^2 + a(t)\left(dx^2 + dy^2 + dz^2\right). \tag{2}$$

Adopting a Friedmann-Robertson-Walker (FRW) metric in 4-dimensions, the Lagrange multiplier for $f(G)$ results [2]

$$G = 24\frac{\dot{a}^2\ddot{a}}{a^3}. \tag{3}$$

Solutions of the $f(G)$ gravity

Nonvacuum case

The action in 4-dimensions will have the following form

$$S = \int dx^4\sqrt{-g}\left[\frac{\varphi(t)f(G)}{2} - \lambda\left(G - 24\frac{\dot{a}^2\ddot{a}}{a^3}\right) + \frac{\dot{\varphi}^2}{2} - V(\varphi)\right], \tag{4}$$

where $V(\varphi)$ – scalar field potential, $\varphi(t)$ – scalar field, λ – Lagrange multiplier. Varying this action with respect to G, we obtain $\lambda = \sqrt{-g}\varphi(t)f'(G)$, $\sqrt{-g} = a^3$, and hense Eq. (4) becomes

$$S=\int dx^4\left[\frac{\varphi(t)f(G)}{2}-a^3\varphi(t)f'(G)\left(G-24\frac{\dot{a}^2\ddot{a}}{a^3}\right)\right]=$$
$$=\int dx^4\left[a^3\,\frac{\varphi(t)f(G)}{2}-a^3\varphi(t)f(G)G+24\dot{a}^2\ddot{a}\varphi(t)f'(G)\right] \tag{5}$$

Replacing in Eq. (5) and integrating by part the Lagrangian finally takes the form:

$$L=a^3\,\frac{\varphi(t)f(G)}{2}-a^3\varphi(t)f'(G)G+8\dot{a}^2\dot{\varphi}(t)f'(G)+8\dot{a}^2\varphi(t)f''(G)\dot{G}\,, \tag{6}$$

$$\frac{\partial L}{\partial q}-\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}}\right)=0\,, \tag{7}$$

$$q=\{a,\varphi,G\};\quad \dot{q}=\{\dot{a},\dot{\varphi},\dot{G}\}.$$

Euler–Lagrange equations (7) lead to

$$-16\frac{\ddot{a}^2}{a^2}\dot{\varphi}(t)f'(G)-16\frac{\dot{a}\ddot{a}}{a^2}\ddot{\varphi}(t)f'(G)-32\frac{\dot{a}\ddot{a}}{a^2}\dot{\varphi}(t)f''(G)\dot{G}-$$
$$-16\frac{\ddot{a}^2}{a^2}\varphi(t)f''(G)\dot{G}-16\frac{\dot{a}\ddot{a}}{a^2}\varphi(t)f'''(G)\dot{G}-16\frac{\dot{a}\ddot{a}}{a^2}\varphi(t)f''(G)\ddot{G}=-3\frac{\varphi(t)f(G)}{2}+3\varphi(t)f'(G)G\,, \tag{8}$$

$$-16\frac{\dot{a}\ddot{a}}{a^2}\,f'(G)=-a\frac{f(G)}{2}+af'(G)G\ \,, \tag{9}$$

$$16\frac{\dot{a}\ddot{a}}{a^2}\varphi(t)f''(G)+8\frac{\dot{a}^2}{a^2}\dot{\varphi}(t)f''(G)+8\frac{\dot{a}^2}{a^2}\varphi(t)f''(G)\dot{G}=-a\varphi(t)f'(G). \tag{10}$$

The system is completed by the Energy function

$$E=\frac{\partial L}{\partial \dot{a}}\dot{a}+\frac{\partial L}{\partial \dot{\varphi}}\dot{\varphi}+\frac{\partial L}{\partial \dot{G}}\dot{G}-L\,, \tag{11}$$

$$E=16\frac{\dot{a}^2\ddot{a}}{a^3}\dot{\varphi}(t)f'(G)+16\frac{\dot{a}^2\ddot{a}}{a^3}\varphi(t)f''(G)\dot{G}-\frac{\varphi(t)f(G)}{2}+\varphi(t)f'(G)G\,. \tag{12}$$

The Noether symmetry approach

In the present case, the vector field of the generator is defined as

$$X=\alpha\frac{\partial}{\partial a}+\beta\frac{\partial}{\partial \varphi}+\gamma\frac{\partial}{\partial G}+\dot{\alpha}\frac{\partial}{\partial \dot{a}}+\dot{\beta}\frac{\partial}{\partial \dot{\varphi}}+\dot{\gamma}\frac{\partial}{\partial \dot{G}}\ \,, \tag{13}$$

on the tangent space $TQ=\{a,\varphi,G,\dot{a},\dot{\varphi},\dot{G}\}$ of the configuration space $Q=\{a,\varphi,G\}$. Here, $\alpha=\alpha(a,\varphi,G)$, $\beta=\beta(a,\varphi,G)$ and $\gamma=\gamma(a,\varphi,G)$ are symmetry generators to be found and

$$\dot{\alpha} = \dot{a}\frac{\partial \alpha}{\partial a} + \dot{\varphi}\frac{\partial \alpha}{\partial \varphi} + \dot{G}\frac{\partial \alpha}{\partial G};$$

$$\dot{\beta} = \dot{a}\frac{\partial \beta}{\partial a} + \dot{\varphi}\frac{\partial \beta}{\partial \varphi} + \dot{G}\frac{\partial \beta}{\partial G}; \tag{14}$$

$$\dot{\gamma} = \dot{a}\frac{\partial \gamma}{\partial a} + \dot{\varphi}\frac{\partial \gamma}{\partial \varphi} + \dot{G}\frac{\partial \gamma}{\partial G}.$$

After the action according to (4), taking into account (6), we explicitly make elementary transformations

$$XL = 0,$$

$$X = \alpha\frac{\partial L}{\partial a} + \beta\frac{\partial L}{\partial \varphi} + \gamma\frac{\partial L}{\partial G} + \dot{\alpha}\frac{\partial L}{\partial \dot{a}} + \dot{\beta}\frac{\partial L}{\partial \dot{\varphi}} + \dot{\gamma}\frac{\partial L}{\partial \dot{G}}. \tag{15}$$

As a result, we obtain a system of equations for symmetry generators

$$3\alpha\varphi(t)\left(\frac{f(G)}{2} - f'(G)G\right) + \beta a\left(\frac{f(G)}{2} - f'(G)G\right) - \gamma a\varphi(t)f'(G) = 0, \tag{16}$$

$$2\frac{\partial \alpha}{\partial a}\dot{\varphi}(t)f'(G)\ddot{a} + \frac{\partial \beta}{\partial \varphi}\dot{\varphi}(t)f'(G) + 2\frac{\partial \alpha}{\partial a}\varphi(t)f''(G)\dot{G}\ddot{a} + \frac{\partial \gamma}{\partial G}\varphi(t)f''(G)\dot{G} = 0, \tag{17}$$

$$2\frac{\partial \alpha}{\partial \varphi}\dot{\varphi}^2(t)f'(G)\ddot{a} + \frac{\partial \beta}{\partial G}\dot{a}f'(G) + 2\frac{\partial \alpha}{\partial \varphi}\varphi(t)f''(G)\dot{G}\dot{\varphi}\ddot{a} + \frac{\partial \gamma}{\partial \alpha}\dot{a}^2\varphi(t)f''(G) = 0, \tag{18}$$

$$\frac{\partial \alpha}{\partial G} = 0, \tag{19}$$

$$\frac{\partial \beta}{\partial a} = 0, \tag{20}$$

$$\frac{\partial \gamma}{\partial \varphi} = 0, \tag{21}$$

where $\alpha, \beta, \gamma, f(G)$ are unknowns to be determined. We solve this set of equations using the method of separation of variables and a power-law form [3].

First, we solve Eqs. (16)–(21) by separation of variables. For this, we set

$$\alpha = A_1(a)A_2(\varphi)A_3(G), \qquad \beta = B_1(a)B_2(\varphi)B_3(G), \qquad \gamma = C_1(a)C_2(\varphi)C_3(G) \tag{22}$$

where $A_1, A_2, A_3, B_1, B_2, B_3$ and C_1, C_2, C_3 are unknowns to be determined. Solving Eqs. (19), (20) and (21) using Eq. (22)

$$\alpha = a_0 A_2(\varphi) A_3(G), \qquad \beta = b_0 B_2(\varphi) B_3(G), \qquad \gamma = c_0 C_2(\varphi) C_3(G) \tag{23}$$

where a_0, b_0 and c_0 are arbitrary constants.

$$A_1(a) A_2(\varphi) = \frac{\lambda}{2a_0} a(t) + a_0, \qquad B_2(\varphi) B_3(G) = -\frac{2\lambda}{b_0}\varphi + b_1, \qquad C_1(a) C_3(G) = -\frac{2\lambda}{c_0} G + c_1. \tag{24}$$

Eq. (24) substitute into Eq. (22)

$$\alpha = A_1(a) A_2(\varphi) A_3(G) = \left(\frac{\lambda}{2a_0} a(t) + a_1\right) a_0,$$

$$\beta = B_1(a) B_2(\varphi) B_3(G) = \left(-\frac{2\lambda}{b_0}\varphi + b_1\right) b_0, \tag{25}$$

$$\gamma = C_1(a) C_2(\varphi) C_3(G) = \left(-\frac{2\lambda}{c_0} + c_1\right) c_0,$$

it follows that

$$\alpha = \frac{\lambda}{2} a(t) + a_0 a_1, \qquad \beta = b_0 b_1 - 2\lambda\varphi, \qquad \gamma = c_0 c_1 - 2\lambda G \tag{26}$$

where a_1, b_1, c_1 – arbitrary constants, λ – separation constant. We use equation (26) by (16) to get

$$f(G) = k\left(-3\frac{\lambda}{2} a(t)\varphi(t) G - 3a_0 a_1 \varphi(t) G - b_0 b_1 a G + 2\lambda\varphi a G - c_0 c_1 a\varphi + 2\lambda G a \varphi(t)\right)^{-1/2}, \tag{27}$$

where k is the constant of integration. Thus, the symmetry generator turns out to be

$$X = \alpha\frac{\partial}{\partial a} + \beta\frac{\partial}{\partial\varphi} + \gamma\frac{\partial}{\partial G} = \left(\frac{\lambda}{2} a(t) + a_0 a_1\right)\frac{\partial}{\partial a} + (b_0 b_1 - 2\lambda\varphi)\frac{\partial}{\partial\varphi} + (c_0 c_1 - 2\lambda G)\frac{\partial}{\partial G}, \tag{28}$$

which obviously corresponds to the scaling symmetry generator. The conserved quantity determined as follows [4]

$$I = \alpha\frac{\partial L}{\partial\dot{a}} + \beta\frac{\partial L}{\partial\dot{\varphi}} + \gamma\frac{\partial L}{\partial\dot{G}} =$$

$$= 8\dot{a}\left[\lambda\ddot{a}a(t)\dot{\varphi}(t) f'(G) + 2\ddot{a}a_0 a_1\dot{\varphi}(t) f'(G) + \lambda\ddot{a}a(t)\varphi(t) f''(G)\dot{G} + 2\ddot{a}a_0 a_1\varphi(t) f''(G)\dot{G}\right] + \tag{29}$$
$$+ 8\dot{a}\left[\dot{a}b_0 b_1 f'(G) - 2\lambda\dot{a}\varphi f'(G) + \dot{a}c_0 c_1\varphi(t) f''(G) - 2\lambda\dot{a}\varphi(t) f''(G) G\right]$$

We use the power law approach to solve the system of Eqs. (16)–(21)

$$\alpha = \alpha_0 a^{\mu_0}\varphi^{\mu_1} G^{\mu_2}, \quad \beta = \beta_0 a^{\upsilon_1}\varphi^{\upsilon_2} G^{\upsilon_2}, \quad \gamma = \gamma_0 a^{\sigma_0}\varphi^{\sigma_1} G^{\sigma_2}, \tag{30}$$

where $\mu_0, \mu_1, \mu_2, \upsilon_0, \upsilon_1, \upsilon_2, \sigma_0, \sigma_1, \sigma_2$ are the unknown powers. Equations (16) and (17) with Eq. (18) imply that $\mu_2 = \upsilon_0 = \sigma_1 = 0$,we obtain

$$\alpha = \alpha_0 a^{\mu_0} \varphi^{\mu_1}, \quad \beta = \beta_0 \varphi^{\upsilon_1} G^{\upsilon_2}, \quad \gamma = \gamma_0 a^{\sigma_0} G^{\sigma_2}. \tag{31}$$

Using these values in (18) with $\alpha_0 = \beta_0 = \gamma_0$ (without any loss of generality), we obtain $\mu_1 = \frac{1}{4}, \nu_2 = 1$ and $\sigma_0 = 1$. Hence, the solution becomes

$$\alpha = \alpha_0 a^{\mu_0} \varphi^{1/4}, \qquad \beta = \alpha_0 \varphi^{\upsilon_1} G, \qquad \gamma = \alpha_0 a G^{\sigma_2}. \tag{32}$$

The function $f(G)$ is obtained by using Eqs. (32) and (16) as

$$f(G) = 2k_1 G^{\left(-\frac{3a^{\mu_0}\varphi^{1/4}\varphi(t)+\varphi^{\upsilon_1}Ga}{3a^{\mu_0}\varphi^{1/4}\varphi(t)+\varphi^{\upsilon_1}Ga+a^2G^{\sigma_2-1}\varphi(t)}\right)}. \tag{33}$$

Where k_1 – another constant of integration. Consequently, the symmetry generator takes the form

$$X = \alpha \frac{\partial}{\partial a} + \beta \frac{\partial}{\partial \varphi} + \gamma \frac{\partial}{\partial G} = \alpha_0 a^{\mu_0} \varphi^{1/4} \frac{\partial}{\partial a} + \alpha_0 \varphi^{\upsilon_1} G \frac{\partial}{\partial \varphi} + \alpha_0 a G^{\sigma_2}, \tag{34}$$

which obviously corresponds to a scaling symmetry generator. The corresponding conserved quantity is

$$I = \alpha \frac{\partial L}{\partial \dot{a}} + \beta \frac{\partial L}{\partial \dot{\varphi}} + \gamma \frac{\partial L}{\partial \dot{G}} =$$

$$= 8\dot{a}\left[2\ddot{a}\alpha_0 a^{\mu_0} \varphi^{1/4} \dot{\varphi}(t) f'(G) + 2\ddot{a}\alpha_0 a^{\mu_0} \varphi^{1/4} \varphi(t) f''(G)\dot{G} + \dot{a}\alpha_0 \varphi^{\upsilon_1} f'(G)G + \dot{a}\alpha_0 a G^{\sigma_2} \varphi(t) f''(G)\right]. \tag{35}$$

Modified theories with their corresponding actions involving higher-order curvature as extra terms can lead to a better description of the phenomenon of cosmic accelerated expansion. In this context, we study solutions of the field equations in $f(G)$ (modified Gauss– Bonnet) gravity using the Noether symmetry approach for the FRW universe model by assuming vacuum and nonvacuum cases. In the nonvacuum case, we have taken dust fluid just for simplicity. The symmetry generators are found using separation of variables and a power-law form. We have found the $f(G)$ model, and the corresponding conserved quantities [5].

This research has been funded by the Science Committee of the Ministry of Education and Science of the Republic of Kazakhstan (Grant No. AP08052034)

References

1. E. J. Copeland, M. Sami, and S. Tsujikawa, Dynamics of dark energy // International Journal Modern Physics D. – 2006. – Vol. 15 № 11. – P. 112.
2. M. Sharif and H. I. Fatima, Dynamics of stellar filaments in f(G) gravity // Astrophysics and Space Science. – 2014. – Vol. 132 № 127. – P. 354.
3. H. K. Jassal, J. S. Bagla, and T. Padmanabhan, Observational constraints on low redshift evolution of dark energy: How consistent are different observations? // Physical Review D. – 2005. – Vol. 72 №67. – P. 135.

4. D. E. Neuenschwander, Emmy Noether's Wonderful Theorem // John's Hopkins University Press, Baltimore. – 2011. – Vol. 9 № 5. – P. 27.
5. B. M. N. Carter and I. P. Neupane, Dynamical relaxation of dark energy: A solution to early inflation, late-time acceleration and the cosmological constant problem // Journal of Cosmology and Astroparticle Physics. – 2006. – Vol. 638 № 58. – P. 94-99.

UDC 517.957, 532.5

MODELING PHYSICAL PROCESSES USING COMSOL MULTIPHYSICS

Temirtas D.M.
temirtasdm@gmail.com
4th year undergraduate student of L.N. Gumilyov Eurasian National University,
Nur-Sultan, Kazakhstan
Scientific supervisor – Shaikhova G.N.

The Modified Korteweg-De Vries equation models a variety of nonlinear phenomena, including ion acoustic waves in plasmas, and shallow water waves. In this research the mKdV equation shall be simulated using Comsol Multiphysics, computer simulation environment where this partial differential equation can be modeled and visualized. Generally, this equation looks the next way:

$$u_t - au^2u_x + u_{xxx} = 0\,. \tag{1}$$

The parameter a can be considered as any real number, where the commonly used values are $a = \pm 1\ or\ a = \pm 6$.

The function $u(x,t)$ represents the water's free surface in non-dimensional variables. The nonlinear KdV equation gives a large variety of solutions. The solutions propagate at speed c while retaining its identity. We usually introduce the new wave variable $\xi = x - ct$, so that $u(x,t) = u(\xi)$.

The derivative ut characterizes the time evolution of the wave propagating in one direction, the nonlinear term uux describes the steepening of the wave, and the linear term $uxxx$ accounts for the spreading or dispersion of the wave. [1]

Obtaining analytical solution of Modified Korteweg-De Vries equation

Integrating once (1) we find, where constant of integration is taken to be zero:

$$-cu + \frac{a}{3}u^3 + u'' = 0. \tag{2}$$

Then we use cosine method to find the solution of the mKdV equation:

$$u = \lambda\, cos^{\beta}(\mu\xi), \tag{3}$$

$$u^3 = \lambda^3\, cos^{3\beta}(\mu\xi), \tag{4}$$

$$u'' = -\mu^2\beta^2\lambda\, cos(\mu\xi) + \mu^2\lambda\beta(\beta-1)cos^{\beta-2}(\mu\xi), \tag{5}$$

$$-c\lambda\, cos^{\beta}(\mu\xi) + \frac{a}{3}\lambda^3\, cos^{3\beta}(\mu\xi) - \mu^2\beta^2\lambda\, cos^{\beta}(\mu\xi) +$$

$$+\mu^2\lambda\beta(\beta-1)cos^{\beta-2}(\mu\xi) = 0\,. \tag{6}$$