having property (N). In [12] was established a sufficient condition for a class **K** to have continuum many subclasses with the property (N) but which are not Q-universal.

In this work we construct a finite modular lattice T that does not satisfies to one of the Tumanov's conditions but quasivariety **Q**(T) generated by this lattice is not finitely based. It has no finite basis of quasi-identities. And then we investigate the topological quasivariety generated by the lattice T and prove that it is not standard.

Theorem 3. The topological quasivariety generated by the lattice T is not standard.

Moreover, the following theorem is true.

Theorem 4. Suppose L is a finite lattice such that the lattices $M_{3,3}$, T, L_n (for all $n>1$) are not sublattices of the lattice L ($M_{3,3}$ and L_n are shown in Figures 2,3,4). Then topological quasivariety generated by the lattice L is not standard.

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UDC 517

LOCALLY-NILPOTENT DERIVATIONS OF THE ALGEBRA OF DIFFERENTIAL POLYNOMIALS AND NOVIKOV ALGEBRAS

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Well known [1, 2, 3, 4], that the automorphisms of the polynomial algebra and the free associative algebra of two variables over an arbitrary field *k* are tame. It is also known that automorphisms of two-generated free Poisson algebras over fields of characteristic zero [5] and automorphisms of two-generated free right-symmetric algebras over arbitrary fields [6] are tame. P. Kohn [7] proved that automorphisms of free Lie algebras of finite rank are tame. Automorphism groups of the algebra of polynomials [8, 9, 10] and free associative algebras [11, 12] of three variables over a field of characteristic zero cannot be generated by all elementary automorphisms, i.e. there are wild automorphisms.

R. Renchler [13] proved that locally nilpotent differentiations of the algebra of polynomials in two variables over a field of characteristic zero are triangulable. Using this result, he gave another proof of Young's theorem [1] on the handedness of automorphisms of these algebras.

Recall that an algebra $\langle N, \circ \rangle$ is called a (left) Novikov algebra, if A satisfies the following identities:

$$(a \circ b) \circ c - a \circ (b \circ c) = (b \circ a) \circ c - b \circ (a \circ c),$$
$$(a \circ b) \circ c = (a \circ c) \circ b,$$

for any $a, b, c \in N$.

In this paper, we investigate locally nilpotent differentiations of the algebra of differential polynomials from two variables with one differentiation and prove that automorphisms of free Novikov algebras from two variables over a field of characteristic zero are tame.

Let be R – an arbitrary commutative ring with unity. The mapping $d : R \to R$ is called *differentiation*, if the conditions are met for all $s, t \in R$:

$$d(s + t) = d(s) + d(t),$$
$$d(st) = d(s)t + s\,d(t).$$

Let $\Delta = \{\delta_1, \ldots, \delta_m\}$ – be the basic set of differential operators. A ring R is called a differential ring or Δ*-ring,* if $\delta_1, \ldots, \delta_m$ are commuting differentiations of the ring R, i.e. $\delta_i : R \to R$ – differentiations and $\delta_i \delta_j = \delta_j \delta_i$ for all i, j. If a ring R is a domain of integrity or a field, then it is called a *differential domain of integrity* or *a differential field.*

Let Θ be a free commutative monoid on the set of differential operators $\Delta = \{\delta_1, \ldots, \delta_m\}$. The elements of a monoid Θ

$$\theta = \delta_1^{i_1} \ldots \delta_m^{i_m}$$

are called *derivative operators.*

Let R be an random differential ring and let $X = \{x_1, \ldots, x_n\}$ be a set of symbols. Consider the set of symbols $X^{\Theta} = \{x_i^{\theta} \mid 1 \le i \le n, \theta \in \Theta\}$ and the algebra of polynomials $R[X^{\Theta}]$ on the set of symbols X^{Θ}. Assuming that

$$\delta_i (x_j^{\theta}) = x_j^{\theta \delta_i}$$

for all $1 \le i \le m$, $1 \le j \le n$, $\theta \in \Theta$, we convert algebra $R[X^{\Theta}]$ into differential algebra. The differential algebra $R[X^{\Theta}]$ is denoted by $R\{X\}$ and is called the algebra of differential polynomials over R from a set of variables X [1].

Let M be a free commutative monoid of a set of variables x_i^{θ}, where $1 \le i \le n$ and $\theta \in \Theta$. The elements are called *monomes* of the algebra $R\{x_1, x_2, \ldots, x_n\}$. Any element $a \in R\{x_1, x_2, \ldots, x_n\}$ is uniquely written in the form

$$a = \sum_{m \in M} r_m m$$

with a finite number of nonzero $r_m \in R$.

Let k be an arbitrary differential field of characteristic 0 and $A = k\{X\} = k\{x_1, x_2, \ldots, x_n\}$ – be the algebra of differential polynomials over the field k of the set of variables X. For any $0 \ne f, g \in A$, we have

$$\alpha(fg) = \alpha(f) + \alpha(g),$$

$$\text{mdeg}(fg) = \text{mdeg}(f) + \text{mdeg}(g),$$

$$\deg(fg) = \deg(f) + \deg(g).$$

We define the degree function with respect to x_i on the algebra A, as $deg_{x_i}\left(x_j^{(s)}\right) = \delta_{ij}$, where $x_j^{\theta} \in X^{\Theta}$, δ_{ij} is the Kronecker symbol and $1 \le i, j \le n$. Homogeneous elements of the algebra A with respect to deg_{xi} are defined in a standard way.

If $f \in A$ is homogeneous with respect to every deg_{xi}, where $1 \le i \le n$, then f is called *multi homogeneous.*

Let $A = k\{x_1, x_2\}$ be the algebra of differential polynomials in variables $x_1, \ldots, x_n$ with one differentiation δ. For the convenience of writing, the derivatives $a^{\delta}, a^{\delta^2}, a^{\delta^s}$ are denoted by. $a', a'', a^{(s)}$, respectively. Let $X = \{x_1, x_2\}$ and denote by X^{δ} the set of all symbols of the form $x_i^{(r)}$,where $i = 1.2$, $r \in Z_+$. For any $D(A_r^*) \subseteq A_{r+t}^*$ we will assume that $x_i^{(r)} > x_j^{(s)}$, if $i < j$ or if $i = j$, $r > s$. The set of all differential monoms of the form

(1) $$u = x_{i_1}^{(s_1)} x_{i_2}^{(s_2)} \ldots x_{i_t}^{(s_t)},$$

where $t \ge 0$, $x_{i_j}^{(s_j)} \in X^{\delta}$ for all $1 \le j \le t$ and $x_{i_1}^{(s_1)} \ge x_{i_2}^{(s_2)} \ge \ldots \ge x_{i_t}^{(s_t)}$, forms the linear basis of the algebra $A = k\{x_1, x_2\}$.

Consider an algebra $Der(A)$, consisting of all the differentiations of an algebra $A = k\{x_1, x_2\}$. For each system of elements $h_1, h_2, \ldots, h_n$ of algebra A we denote by

(2) $$D = h_1\partial_1 + h_2\partial_2$$

the differentiation of an algebra A is such that , $D(x_1) = h_1$, $D(x_2) = h_2$.

Then differentiation of the form

(3) $$v = u\partial_i,$$

where $i = 1, 2$ and u are an element of the form (1), constitute a linear basis $Der(A)$. For each element v of the form (3) we put

$$\deg(v) = \deg(u) - 1.$$

The following statement is well known.

Proposition 1. *Let* $R=\oplus_{m\in Z}R_m$ *– градуированная алгебра. be a graded algebra. Suppose that D is a locally nilpotent differentiation of an algebra R such that*

$$D=D_p+D_{p+1}+\ ...+D_q\ ,\quad D_i\left(R_m\right)\subseteq R_{i+m}\ ,p\le i\le q\ ,\ D_q\ne 0$$

Then D_q *is locally nilpotent.*

Differentiation of the form (2) of the algebra $k\{x_1,...,x_n\}$ is called *triangular*, if $h_i\in k\{x_{i+1},...,x_n\}$ *for any* $i<n$ *and* $h_n\in k$.

Recall that in the derivation D of the algebra R is called locally nilpotent if for any $a\in R$ exists a positive integer $m=m(a)$ *where* $D^m(a)=0.$

Statement 2. *Any triangular differentiation of the algebra* $k\{x_1,...,x_n\}$ *is locally nilpotent.*

Proposition 3. *Let D be the differentiation of the algebra A and x be the minimal element of* $S(D)$. *Then*

$$deg_x D(f)\le deg_x D+deg_x f\ .$$

This inequality becomes equality if and only if $l_x(D)(l_x(f))\ne 0$ *and in this case*

$$l_x(D(f))=l_x(D)(l_x(f)).$$

Lemma 1. *Let* D *be the differentiation of the algebra A and x be the minimal element* $S(D)$. *If* D *locally-nilpotent differentiation, then* $l_x(D)$ *also locally-nilpotent.*

Lemma 2. *Let* D *be the differentiation of an algebra A of the form*

$$D=x_{i_1}^{(s_1)}x_{i_2}^{(s_2)}\ldots x_{i_r}^{(s_r)}\partial_j\ .$$

where $x_{i_1}^{(s_1)}\ge x_{i_2}^{(s_2)}\ge\ldots\ge x_{i_r}^{(s_r)}$. *If there is at least one* k, $1\le k\le r$ *, such that* $j=i_k$, *then* D *is not a locally nilpotent differentiation.*

Lemma 3. *Let* D *be a multi–homogeneous differentiation of the algebra* $A=k\{x_1,x_2\}$ *with a multidegree* $mdeg(D)=(m_1,m_2)$. *If* $m_i\ge 0$ *for* $i=1,2$ *, then* D *is not a locally nilpotent differentiation.*

Lemma 4. *Differentiation* D *of the algebra* $A=k\{x_1,x_2\}$ *of the form*

$$D=f_1(x_2)\partial_1+f_2(x_1)\partial_2\ ,$$

where $f_1(x_2)\in k\{x_2\}\setminus k$ *u* $f_2(x_1)\in k\{x_1\}\setminus k^*$, *is locally nilpotent if and only if* $f_2(x_1)=0$.

Statement 4. *Let D be the differentiation of the algebra A and x be the minimal element of* $S(D)$. *Then*

$$pdeg_x D(f)\le pdeg_x D+pdeg_x f\ .$$

This inequality becomes equality if and only if $l_x(D)(l_x(f))\ne 0$ *and in this case*

$$l_x(D(f))=l_x(D)(l_x(f)).$$

Theorem 1. Let D be a locally nilpotent differentiation of an algebra $N\langle x_1,x_2\rangle$. Then there exists a manual automorphism ϕ of the algebra $N\langle x_1,x_2\rangle$ and $g(x_2)\in N\langle x_2\rangle$ such that $\varphi^{-1}D\varphi=g(x_2)\partial_1$.

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BOUNDEDNESS OF THE MAXIMAL OPERATOR IN THE WEIGHTED LOCAL MORREY-LORENTZ SPACES

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