List of used sources

1. Jung H.W.E. Uber ganze birationale Transformationen der Ebene // J. reine angew. Math. – 1942. – Vol. 184. – P. 161-174.
2. Kulk W. Van der. On Polynomial Rings in Two Variables // Nieuw Archief voor Wiskunde. – 1953. – Vol. 3, № 1. – P. 33-41.
3. Czerniakiewicz A.G. Automorphisms of a Free Associative Algebra of Rank 2. I, II // Trans. Amer. Math. Soc. – 1971. – Vol. 160. – P. 393-401; – 1972. – Vol. 171. – P. 309-315.
4. Макар-Лиманов Л. Автоморфизмы свободной алгебры от двух порождающих // Функцион. анализ и его прил. – 1970. – Т. 4. – С. 107-108.
5. Makar-Limanov L., Turusbekova U., Umirbaev U.U. Automorphisms and derivations of free Poisson algebras in two variables // J. Algebra. – 2009. – Vol. 322, № 9. – P. 3318-3330.
6. Kozybaev D., Makar-Limanov L., Umirbaev U. The Freiheitssatz and the automorphisms of free right-symmetric algebras // Asian-European Journal of Mathematics. – 2008. – Vol. 1. – P. 243-254.
7. Cohn P.M. Subalgebras of free associative algebras // Proc. London Math. Soc. – 1964. – Vol. 56. – P. 618-632.
8. Shestakov I.P., Umirbaev U.U. The Nagata automorphism is wild // Proc. Natl. Acad. Sci. USA. – 2003. – Vol. 100, № 22. – P. 12561-12563.
9. Shestakov I.P. and Umirbaev U.U. Tame and wild automorphisms of rings of polynomials in three variables // J. Amer. Math. Soc. – 2004. – Vol. 17. – P. 197-227.
10. Шестаков И.П., Умирбаев У.У. Подалгебры и автоморфизмы колец многочленов // Докл. РАН. – 2002. – Т. 386, № 6. – С. 745-748.
11. Умирбаев У.У. Определяющее соотношения группы ручных автоморфизмов алгебры многочленов и дикие автоморфизмы свободных ассоциативных алгебр // Докл. РАН. – 2006. – Т. 407, № 3. – С. 319-324.
12. Umirbaev U.U. The Anick automorphism of free associative algebras // J. Reine Angew. Math. – 2007. – Vol. 605. – P. 165-178.
13. Rentschler R. Operations du groupe additif sur le plan // C.R. Acad. Sci. Paris. – 1968. – Vol. 267. – P. 384-387.
14. Kolchin E.R. Differential Algebra and Algebraic Groups. Pure and Applied Mathematics, 54. Academic Press, New York-London, 1973.
15. Van den Essen A. Polynomial automorphisms and the Jacobian conjecture, Progress in Mathematics, 190, Birkhauser verlag, Basel, 2000
16. Дуйсенгалиева Б.А., Умирбаев У.У. Дикий автоморфизм свободной алгебры Новикова // Сибирские электронные математические известия. – 2018. – Т. 15. – С. 1671-1679.
17. Дуйсенгалиева Б.А., Науразбекова А.С., Умирбаев У.У. Ручные и дикие автоморфизмы алгебры дифференциальных многочленов ранга 2 // Фундаментальная и прикладная математика. – 2019. – Т. 22, № 4. – С. 101-114.

UDC 517.52

BOUNDEDNESS OF THE MAXIMAL OPERATOR IN THE WEIGHTED LOCAL MORREY-LORENTZ SPACES

Yelubay Nurdaulet

nurdaulet.yelubay@alumni.nu.edu.kz

PhD student, L.N. Gumilyov Eurasian National University, Nur-Sultan, Kazakhstan

Research supervisors – N.A. Bokayev, M.L.Goldman

The article presents a theorem on the boundedness of the maximal operator in the weighted local Morrey-Lorentz spaces.

Let E be a measurable subset of $\mathbb{R}^n$ and $L_p(E)$ be a set of all measurable functions f defined on E for which the quasi-norm is finite [1]:

$$\|f\|_{L_p(E)} := \left(\int_E |f(y)|^p dy\right)^{\frac{1}{p}} < \infty, \ \ 0 < p < \infty,$$
$$\|f\|_{L_\infty(E)} := \sup\{\alpha : |\{y \in E : |f(y)| \geq \alpha\}| > 0\}.$$

We define decreasing rearrangement of f by

$$f^*(t) = \inf\{\lambda > 0 : \mu_f(\lambda) \leq t\}, \quad t \in R_+,$$

Where $\mu_f(\lambda)$ denotes the distribution function of f given by

$$\mu_f(\lambda) = |\{y \in R^n : |f(y)| > \lambda\}|, \quad (1 \leq p < \infty, 0 < \lambda \leq u)$$

We denote by $L_{p,\lambda}(\mathbb{R}^n)$ the Morrey space for $0 \leq \lambda \leq n, 1 \leq p < \infty, f \in L_p^{loc}(\mathbb{R}^n)$ for which the quasi-norm is finite:

$$\|f\|_{L_{p,\lambda}} = \sup_{x \in R^n, t > 0} t^{-\frac{\lambda}{p}} \|f\|_{L_p(B(x,t))} < \infty,$$

Where $B(x,r)$is a ball with the center at the point x and the radius r.

The Lorentz space $L_{\psi,q}(\mathbb{R}^n), 1 < q \leq \infty, \psi \in \mathcal{M}^+(0,\infty)$, is the collection of all measurable functions f on $\mathbb{R}^n$such the quantity

$$\|f\|_{L_{\psi,q,\lambda}} = \left\| t^{\frac{1}{p}-\frac{1}{q}} f^*(t) \right\|_{Lq(0,\infty)} < \infty.$$

The function $f^{**}: (0,\infty) \to [0,\infty]$ is defined as

$$f^{**}(t) = \frac{1}{t}\int_0^t f^*(s)ds$$

We denote by $\mathfrak{M}(\mathbb{R}^n)$ the set of all extended real-valued measurable functions on $\mathbb{R}^n$ and by $\mathfrak{M}^+(0,\infty)$ the set of all non-negative measurable functions on $(0,\infty)$.

Definition 1:

Let $1 < p \leq \infty$ and $\psi \in \mathfrak{M}^+(0,\infty)$. We denote by $\Lambda_{p,\psi}(\mathbb{R}^n)$ weighted Lorentz spaces, the spaces of all measurable functions with a finite quasi-norm:

$$\Lambda_{p,\psi}(R^n) := \left\{ f \in M(R^n) : \|f\|_{\Lambda_{p,\psi}} := \|\psi f^*\|_{L_p(0,\infty)} \right\}.$$

We denote by $\mathfrak{M}^+((0,\infty),\downarrow)$ the set of all decreasing functions $f \in \mathfrak{M}^+(0,\infty)$.

Definition 2:

Let $1 \leq p,q \leq \infty$ and $0 < \lambda < 1, \ \psi \in \mathcal{M}^+(0,\infty)$. We denote by $M_{p,q,\lambda,\psi}^{loc}(\mathbb{R}^n)$ the weighted local Morrey-Lorentz spaces, the spaces of all measurable functions with a finite quasi-norm:

$$\|f\|_{M_{p,q,\lambda,\psi}^{loc}} := \sup t^{-\frac{\lambda}{q}} \|\psi(s) f^*(s)\|_{L_q(0,t)}.$$

$$\| f \|_{M^{loc}_{q,\lambda,\psi}} = \sup t^{-\frac{\lambda}{q}} \left(\int_0^t \psi(s) f^*(s)^q \right)^{\frac{1}{q}}.$$

The Hardy-Littlewood maximal operator Mf of f is defined by

$$Mf(x) = \sup_{t>0} \frac{1}{|B(x,t)|} \int_{B(x,t)} |f(y)| dy, \ x \in R^n.$$

Where $B(x,r)$is a ball with the center at the point x and the radius r.
The fractional maximal operator $M_\beta f(x)$ is defined by

$$M_\beta f(x) = \sup_{t>0} |B(x,t)|^{\frac{\beta}{n}-1} \int_{B(x,t)} |f(y)| dy, \ x \in R^n.$$

Lemma 1. $\|\cdot\|_{M^{loc}_{p,q,\lambda,\psi}}$ is a quasi-norm on $M^{loc}_{p,q,\lambda}(R^n)$.

Lemma 2. Let $0 < p, q < \infty$ and $0 \le \lambda \le 1$. Then $M^{loc}_{p,q,\lambda,\psi}(R^n) \hookrightarrow L_{p,q,n\lambda\psi}(R^n)$.

Definition 3:

The Lorentz-Morrey spaces $L_{p,q,\lambda}(\mathbb{R}^n)$ is the set of all measurable functions f on $\mathbb{R}^n$: for $1 \le p < \infty$, $0 < q < \infty$ and $0 \le \lambda \le n$, iff

$$\|f\|_{L_{p,q,\lambda}} = sup_{x\in\mathbb{R}^n, t>0} t^{-\frac{\lambda}{q}} \left\| \chi_{B(x,t)} f \right\|_{L_{p,q}} < \infty.$$

Accordingly, f belongs to

$$L_{p,\infty,\lambda}(\mathbb{R}^n) \equiv WL_{p,\lambda}(\mathbb{R}^n) \text{ iff } \|f\|_{L_{p,\infty,\lambda}} = \|f\|_{WL_{p,\lambda}} < \infty.$$

Note that the spaces $\mathcal{L}_{p,q,\lambda}(\mathbb{R}^n)$ and $L_{p,q,\lambda,\frac{q}{p}}(\mathbb{R}^n)$ coincide, thus

$$\mathcal{L}_{p,q,\lambda}(\mathbb{R}^n) = L_{p,q,\lambda,\frac{q}{p}}(\mathbb{R}^n)$$

Theorem 1. Let $1 \le q \le \infty$, $0 < \lambda \le 1$, $\psi \in \mathfrak{M}^+\big((0,\infty),\downarrow\big)$, then the maximal operator M is bounded on the local Morrey-Lorentz spaces $M^{loc}_{\psi,q,\lambda}(\mathbb{R}^n)$.

In case $\psi(t) = t^{\frac{1}{p}-\frac{1}{q}}$, $1 \le p, q \le \infty$ the abovementioned theorem is proven in [2].

Theorem 2. Let $1 < p < q < \infty$, $\beta = \frac{1}{p} - \frac{1}{q}$, $0 < \lambda \le 1$. Then the fractional maximal operator $M_\beta f(x)$ is bounded on the local Morrey-Lorentz spaces $M^{loc}_{p,q,\lambda}$.

References

1. I.Stein, G.Veis, Introduction to harmonical analysis on Euclidian spaces, -M.:Mir, 1974, p.336.
2. C.Aykol, V.Guliyev, A.Serbetci, Boundedness of the maximal operator in the local Morrey-Lorentz spaces // Journal of Inequalities and Applications, 2013, p.1-11.