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METHODS FOR IN-DEPTH STUDY OF INEQUALITIES IN SCHOOL MATHEMATICS

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Modern development of science and technology requires deep knowledge, development of mental abilities and creative work. She is closely connected with the deep education and upbringing of students at school. In particular, the task of each teacher is the deepening of the knowledge of students in mathematics.

One of the topics that influence the fulfillment of such requirements in mathematics is inequalities and how to prove them. Inequalities and how to prove them play an important role in modern mathematics. Inequalities are often used in research, and the results of such research are also reflected in the inequalities.

In this paper methods for proving inequalities by the Cauchy-Bunyakovsky inequality method, proving inequalities by introducing variables, proving inequalities using properties of functions, and proving inequalities by mathematical induction are considered and grouped. To study the skills and abilities of students that is formed in the process of proving functional inequalities in classes with in-depth study of mathematics.

Such inequalities, which have signs for determining the abilities and capabilities of students and the level of their intellectual development, are considered in mathematics only in classes with in-depth study of mathematics. However, the problems of proving inequalities by these methods are not sufficiently represented in the program.

In school mathematics, these inequalities are used to prove various inequalities. The study of equations, approximate calculations, the theory of irrational numbers, number series, etc. based on the properties of inequalities. In secondary school, in the course of mathematical analysis, inequalities are widely used in solving maximum and minimum functions, i.e., extremal problems.

Continuous processes of nature, studied not only in mathematics, but also in various natural sciences, especially environmental, economic, etc. Relations in the national economy are resolved through inequality. Inequalities teach students to think clearly and correctly, to compare values.

The analysis of such problems requires from students a deep search, thinking, intellectual development, contributes to the formation of their skills and abilities.

Consider ways to solve some inequalities using the Cauchy-Bunyakovsky inequality.

Firstly we prove for numbers a_1, a_2, b_1, b_2.

Let $\vec{a}(a_1, a_2)$, $\vec{b}(b_1, b_2)$ be vectors. We know the following formula from a school course

$$a \cdot b = a_1b_1 + a_2b_2 = |\vec{a}| \cdot |\vec{b}| \cos\left(\vec{a} \cdot \vec{b}\right)$$

Estimate the modulus of the scalar product $\vec{a}, \vec{b}$

$$\left|\vec{a} \cdot \vec{b}\right| = |\vec{a}| \cdot |\vec{b}| \left|\cos\left(\vec{a};\vec{b}\right)\right| \le |\vec{a}| \cdot |\vec{b}|.$$

Secondly

$$\left|\vec{a} \cdot \vec{b}\right| = |a_1b_1 + a_2b_2| \le |\vec{a}| \cdot |\vec{b}| \left|\cos\left(\vec{a};\vec{b}\right)\right| = \sqrt{a_1^2 + a_2^2} \cdot \sqrt{b_1^2 + b_2^2},$$

or

$$\left(a_1b_1 + a_2b_2\right)^2 \le (a_1^2 + a_1^2)(b_1^2 + b_2^2). \qquad (1)$$

This is a special case of the Cauchy-Bunyakovsky inequality for numbers a_1, a_2, b_1, b_2.

Fulfillment of equality $a_1b_2 - a_2b_1 = 0$ is necessary and sufficient for fulfillment of inequality.

The generalized inequality (1) is also called the Cauchy-Bunyakovsky inequality for numbers $a_1, a_2, ... a_n, b_1, b_2, ..., b_n$, and it is given in the following form:

$$\left(a_1^2 + a_2^2 + ... + a_n^2\right)\left(b_1^2 + b_2^2 + ... + b_n^2\right) \ge \left(a_1b_1 + a_2b_2 + ... + a_nb_n\right)^2 \qquad (2)$$

We prove inequality (2) for $a_1, a_2, ... a_n, b_1, b_2, ..., b_n \ge 0$.

Let

$$x_k = \sqrt{(a_1^2 + \ldots + a_n^2)(b_1^2 + \ldots + b_n^2} \ ,$$

where $k = 1,2,\ldots,n$. In this case

$$x_{k+1} = \sqrt{a_1^2 + \ldots + a_k^2 + a_{k+1})(b_1^2 + \ldots + b_k^2 + b_{k+1}} =$$

$$\sqrt{\left(\left(\sqrt{a_1^2 + \ldots + a_k^2}\right)^2 + a_{k+1}^2\right)\left(\left(\sqrt{b_1^2 + \ldots + b_k^2}\right)^2 + b_{k+1}^2\right)} \geq$$

$$\geq \sqrt{\left(\sqrt{a_1^2 + \ldots + a_k^2} \cdot \sqrt{b_1^2 + \ldots + b_k^2} + a_{k+1} \cdot b_{k+1}\right)^2} = x_k + a_{k+1} \cdot b_{k+1}$$

Thus, we obtain the following inequality

$$x_{k+1} \geq x_k + a_{k+1} \cdot b_{k+1},$$

where $k = 1,2,\ldots,n-1$.

We get the following inequality

$$\sqrt{(a_1^2 + \ldots + a_n^2)(b_1^2 + \ldots + b_n^2} \geq a_1 b_1 + \ldots + a_n b_n$$

or,

$$\left(a_1^2 + \ldots + a_n^2\right)\left(b_1^2 + \ldots + b_n^2\right) \geq \left(a_1 b_1 + \ldots + a_n b_n\right)^2.$$

Now we prove for any real numbers $a_1, a_2, \ldots a_n, b_1, b_2, \ldots, b_n$

$$\left(a_1^2 + \ldots + a_n^2\right)\left(b_1^2 + \ldots + b_n^2\right) = \left(\left|a_1^2\right| + \ldots + \left|a_n^2\right|\right)\left(\left|b_1^2\right| + \ldots + \left|b_n^2\right|\right) \geq$$

$$\geq \left(\left|a_1 b_1\right| + \ldots + \left|a_n b_n\right|\right)^2 \geq \left|a_1 b_1 + \ldots + a_n b_n\right|^2 = \left(a_1 b_1 + \ldots + a_n b_n\right)^2$$

Consider an example of this method:

If $a > 0, b > 0, c > 0$, then prove the following inequality

$$\frac{1}{\sqrt{bc}} + \frac{1}{\sqrt{ca}} + \frac{1}{\sqrt{ab}} \leq \frac{1}{a} + \frac{1}{b} + \frac{1}{c}.$$

Proof. Consider the left side of the first inequality

$$\frac{1}{\sqrt{b}\sqrt{c}} + \frac{1}{\sqrt{c}\sqrt{a}} + \frac{1}{\sqrt{a}\sqrt{b}}$$

We use the Cauchy-Bunyakovsky inequality $a_1 b_1 + a_2 b_2 + a_3 b_3 \leq \sqrt{a_1^2 + a_2^2 + a_3^2} \cdot \sqrt{b_1^2 + b_2^2 + b_3^2}$

Then we get the following inequality

$$\frac{1}{\sqrt{b}}\frac{1}{\sqrt{c}}+\frac{1}{\sqrt{c}}\frac{1}{\sqrt{a}}+\frac{1}{\sqrt{a}}\frac{1}{\sqrt{b}}\leq\sqrt{\left(\frac{1}{b}+\frac{1}{c}+\frac{1}{a}\right)\left(\frac{1}{c}+\frac{1}{a}+\frac{1}{b}\right)}=\frac{1}{a}+\frac{1}{b}+\frac{1}{c}$$

The prove is complete.

To prove some inequalities, the goal can be achieved by introducing a new variable. Consider the first example of this method.

Prove the inequality

$$-\frac{1}{2}\leq\frac{(x+y)(1-xy)}{(1+x^2)(1+y^2)}\leq\frac{1}{2}.$$

Proof. Let us introduce the notation

$$x=tg\alpha,\ y=tg\beta$$

In this case we get

$$\frac{(x+y)(1-xy)}{(1+x^2)(1+y^2)}=\frac{(tg\alpha+tg\beta)(1-tg\alpha tg\beta)}{(1+tg^2\alpha)(1+tg^2\beta)}=\frac{\frac{\sin(\alpha+\beta}{\cos\alpha\cos\beta}\frac{\cos(\alpha+\beta)}{\cos\alpha\cos\beta}}{\frac{1}{\cos^2\alpha}\frac{1}{\cos^2\beta}}=$$

$$=\sin(\alpha+\beta)\cos(\alpha+\beta)=\frac{1}{2}\sin 2(\alpha+\beta)$$

$$-\frac{1}{2}\leq\frac{1}{2}\sin 2(\alpha+\beta)\leq\frac{1}{2}$$

The prove is complete.

Consider the second example of this method:

If $a,b,c>0,\quad abc=1,$ then prove the following inequality

$$\frac{1}{1+a+b}+\frac{1}{1+b+c}+\frac{1}{1+a+c}\leq 1$$

Proof.

$$\sqrt[3]{a}=x,\sqrt[3]{b}=y\sqrt[3]{c}=z$$

In this case

$$xyz=1,\frac{1}{1+a+b}=\frac{xyz}{xyz+x^3y^3}\leq\frac{xyz}{xyz+xy(x+y)^2}=\frac{z}{x+y+z};$$

We obtain the following inqualities

$$\frac{1}{1+b+c}\leq\frac{x}{x+y+z}\text{ және }\frac{1}{1+a+c}\leq\frac{y}{x+y+z}.$$

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Then

$$\frac{1}{1+a+b}+\frac{1}{1+b+c}+\frac{1}{1+a+c}\leq\frac{x}{x+y+z}+\frac{y}{x+y+z}+\frac{z}{x+y+z}=1$$

The prove is complete.

Consideration of methods for proving inequality is now necessary when entering higher educational institutions, in preparation for the Olympics.

Mastering the methods of proving inequalities, mastering many methods on these methods can be considered as a criterion for the level of knowledge of the main sections of school mathematics, mathematical and logical thinking.

The considered methods have a great influence on the formation of logical thinking and mathematical culture of students.

In conclusion, the following skills and abilities of students are studied, formed in the process of proving functional inequalities in the classes for in-depth study of mathematics: the ability to apply the knowledge gained in previous classes in the process of transformation; deep understanding and memory in the application of acquired knowledge; be able to effectively use the time allotted for each task; striving to get a very good grade; the ability to choose the most efficient way to accomplish a task.

Literature

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