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METHODOLOGY OF GIVING DIFFERENTIAL AND INTEGRAL EQUATIONS BY USING THEIR COMMUNICATIONS AND APPLICATIONS

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Although the concept of differential equations began to be taught in high school, but it is clear that, this topic is difficult for many students, as in university too. It is known that it has their difficulties to understand differential equations, to distinguish their types and to find solutions. Moreover, integral equations are a completely new material for students. In this paper, we try to connect these two equations.

Ordinary differential and integral equations are mathematical models for many applied problems, in particular, in such areas as electrodynamics and elasticity theory.

Since some integral equations can be solved by reducing them to Cauchy problems for ordinary differential equations.

Students can be shown that the Cauchy problem for the linear differential equations can be solved by reducing to an equivalent integral equation. For example, the solution of the linear differential equation

$$\frac{d^n y}{dx^n} + a_1(x)\frac{d^{n-1}y}{dx^{n-1}} + \cdots + a_n(x)y = F(x)$$

with continuous coefficients $a_1(x) \quad (i = 1,2, \dots, n)$ under the initial conditions

$$y(0) = c_0, \qquad y'(0) = c_1, \dots, \qquad y^{n-1}(0) = c_{n-1}$$

can be reduced to the solution of the Volterra integral equation of the second kind.
Let 's show this by the example of a second-order differential equation

$$\frac{d^2 y}{dx^2} + a_1(x)\frac{dy}{dx} + a_2(x)y = F(x) \tag{1}$$

$$y(0) = c_0, \; y'(0) = c_1. \tag{2}$$

We assume that

$$\frac{d^2 y}{dx^2} = \varphi(x). \tag{3}$$

Hence, taking into account the initial conditions (2), we can consistently find

$$\frac{dy}{dx} = \int_0^x \varphi(t)dt + C_1, \qquad y = \int_0^x (x-t)\varphi(t)dt + C_1 x + C_0. \tag{4}$$

At the same time , we used the formula

$$\underbrace{\int_{x_0}^x dx \int_{x_0}^x dx \dots \int_{x_0}^x f(x)dx}_{(n\ times)} = \frac{1}{(n-1)!}\int_{x_0}^x (x-z)^{n-1} f(z)dz.$$

Using (3) and (4), we write the differential equation (1) as follows:

$$\varphi(x) + \int_0^x a_1(x)\varphi(t)dt + C_1 a_1(x) +$$
$$+ \int_0^x a_2(x)(x-t)\varphi(x)dt + C_1 x a_2(x) + C_0 a_2(x) = F(x),$$

or

$$\varphi(x) + \int_0^x [a_1(x) + a_2(x)\,(x-t)]\varphi(t)dt =$$

$$= F(x) - C_1 a_1(x) - C_1 x a_2(x) - C_0 a_2(x), \tag{5}$$

and assuming

$$K(x,t) = -[a_1(x) + a_2(x)(x-t)],$$

$$f(x) = F(x) - C_1 a_1(x) - C_1 x a_2(x) - C_0 a_2(x)$$

We write (5) by the following form

$$\varphi(x) = \int_0^x K(x,t)\varphi(t)dt + f(x),$$

So, we lead Cauchy problem (1) - (2) to the Volterra integral equation of the second kind.

Let us show that the integral equation of Volterra can also be solved using the differential equation when we use a resolvent. The solution of the equation by the resolvent is written as follows

$$\varphi(x) = f(x) + \lambda \int_0^x R(x,t,\lambda) f(t)dt,$$

where

$$R(x,t,\lambda) = \sum_{v=0}^{\infty} \lambda^v K_{v+1}(x,t)$$

is resolvent. If the kernel $K(x,t)$is a polynomial of the *(n-1)*th degree with respect to *t*, so that it can be represented as

$$K(x,t) = a_0(x) + a_1(x) + \ldots + \frac{a_{n-1}(x)}{(n-1)!}(x-t)^{n-1},$$

moreover, the coefficients $a_k(x)$ are continuously in $[0,a]$. If we define the function $g(x,t;\lambda)$ as the solution of the differential equation

$$\frac{d^n g}{dx^n} - \lambda\left[a_0(x)\frac{d^{n-1}g}{dx^{n-1}} + a_1(x)\frac{d^{n-2}g}{dx^{n-2}} + \ldots + a_{n-1}(x)g\right] = 0$$

Satisfying the conditions

$$g_{|z=t} = \left.\frac{dg}{dx}\right|_{x=t} = \ldots = \left.\frac{d^{n-2}g}{dx^{n-2}}\right|_{x=t} = 0, \qquad \left.\frac{d^{n-1}g}{dx^{n-1}}\right|_{x=t} = 1, \qquad (6)$$

Then the resolvent $R(x,t;\lambda)$ will be determined by the equality

$$R(x,t;\lambda) = \frac{1}{\lambda}\frac{d^n g(x,t;\lambda)}{dx^n}$$

Similarly, in the case when

$$K(x,t) = b_0(x) + b_1(x) + \ldots + \frac{b_{n-1}(x)}{(n-1)!}(x-t)^{n-1},$$

Resolvent will be

$$R(x,t;\lambda) = -\frac{1}{\lambda}\frac{d^n g(t,x;\lambda)}{dt^n},$$

where $g(x,t;\lambda)$ there is a solution to the equation

$$\frac{d^n g}{dt^n} + \lambda\left[b_0(t)\frac{d^{n-2}g}{dt^{n-1}} + \ldots + b_{n-1}(t)g\right] = 0,$$

satisfying the conditions (6).

Literature

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