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Correspondence of cosmology from non-extensive thermodynamics with fluids of generalized equation of state

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Abstract

We show that there is a correspondence between cosmology from non-extensive thermodynamics and cosmology with fluids of redefined and generalized equation of state. We first establish the correspondence in the case of basic non-extensive thermodynamics, and then we proceed by investigating the more consistent case, from the quantum field theoretical point of view, of varying exponent, namely depending on the scale. The obtained duality provides a way of explaining the complicated phenomenological forms of

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the effective fluid equation-of-state parameters that are being broadly used in the literature, since their microphysical origin may indeed lie in the non-extensive thermodynamics of spacetime. Finally, concerning the cosmological behavior, we show that at late times the effective fluid may drive the universe acceleration even in the absence of an explicit cosmological constant, and even if the initial fluid is the standard dust matter one. Similarly, at early times we obtain an effective cosmological constant which is enhanced through screening, and hence it can drive a successful inflation without spoiling the correct late-time acceleration.

1. Introduction

In the recent years there has appeared an increasing amount of cosmological data, from early, intermediate and late times, whose successful explanation may imply the need of modifying our current knowledge. The modification of gravity (for general reviews see [1–3]), is one of the two ways that is being followed in order to obtain the extra degrees of freedom required, with the second one being the introduction of the inflaton [4] and/or dark energy fields/fluids [5,6].

Amongst the various approaches to modified gravity one can find the interesting one that is based on the connection between gravity and thermodynamics [7–9]. In such a consideration one applies the first law of thermodynamics in the universe horizon and results to the Friedmann equations, and it proves to be true in various classes of modified gravity [10–22]. In this thermodynamical approach one needs to apply the entropy expression of the involved theory. Although the standard Boltzmann-Gibbs additive entropy is the one that it is usually used, the fact that in the case of non-additive systems, such as large-scale gravitational ones, such an entropy should be generalized to the non-extensive Tsallis entropy [23–25], led to the thermodynamic considerations of gravity based on the latter [26–36]. Indeed, parametrizing the non-extensivity by a new exponent δ, for which the value 1 corresponds to the standard entropy, one may obtain modified Friedmann equations, which may lead to various (old and new) cosmological scenarios compatible with observations [29]. This approach has been further generalized to the non-extensive thermodynamics in which the involved exponent presents a running behavior [37], which is typical for quantum field theory when renormalization group is incorporated. Cosmology from extended entropy with varying exponent is able to provide a successful description of the universe evolution at both late and early times.

On the other hand, it is known that one can describe the early and late time universe evolution by considering perfect fluids with negative pressure, satisfying a barotropic equation of state [38–60] or presenting generalized equation-of-state parameters [61–63] (for a review see [64]). In such scenarios the fluids are considered to arise effectively, without the need of explaining their microphysical origin.

In the present work we show that there is a correspondence between cosmology from non-extensive thermodynamics and cosmology with fluids of redefined and generalized equation of state. This form of duality may be important in providing the unknown microphysical origin of the effective fluids that are used broadly in cosmology. Additionally, we present how different epochs of the universe evolution can be successfully described.

The plan of the work is as follows: In Section 2 we demonstrate the correspondence between cosmology from non-extensive thermodynamics with fluids of generalized equation of state, starting from the former and resulting to the latter, as well as starting from the latter and resulting to the former. In Section 3 we present the same analysis in the extended case where the non-extensive exponent presents a varying behavior, depending on the scale, as it is the case of a quantum field theory when renormalization group is incorporated. Moreover, we apply the scenario at both early- and late-time universe. We close this work with our conclusions in Section 4.

2. Correspondence of non-extensive thermodynamics with fluids of redefined equation of state

In this section we show how cosmology with fluids with redefined equation-of-state parameters can be obtained from non-extensive thermodynamics and vice versa.

2.1. Fluids of redefined equation of state through non-extensive thermodynamics

We start by briefly reviewing how modified cosmology can arise through the application of non-extensive, Tsallis thermodynamics. We apply a homogeneous and isotropic flat Friedmann-Robertson-Walker (FRW) geometry with metric

$$ds^2 = -dt^2 + a(t)^2 \sum_{i=1,2,3} \left(dx^i\right)^2 , \tag{1}$$

with $a(t)$ the scale factor. Moreover, we consider the universe to be filled with a perfect fluid, with energy density and pressure ρ and p respectively. Such a system may be considered as a thermodynamical system bounded by the apparent horizon r_H defined using the Hubble rate $H \equiv \dot{a}/a$ as [10,65]

$$r_H = \frac{1}{H} . \tag{2}$$

The energy dE going outward through the sphere of radius r_H in a time interval dt is given by [10]

$$dQ = -dE = \frac{4\pi}{3} r_H^3 \dot{\rho} dt = \frac{4\pi}{3H^3} \dot{\rho} dt . \tag{3}$$

By using the standard conservation law

$$0 = \dot{\rho} + 3H(\rho + p) , \tag{4}$$

one can rewrite Eq. (3) as

$$dQ = \frac{4\pi}{H^2} (\rho + p) dt . \tag{5}$$

The next step is to introduce the entropy through

$$TdS = dQ , \tag{6}$$

with the Hawking temperature T given by [10,65]

$$T = \frac{1}{2\pi r_H} = \frac{H}{2\pi} . \tag{7}$$

If the usual entropy relation $S = \frac{A}{4G}$ (with $A = 4\pi r_H^2$ the horizon area) is used then one can immediately see that (5), (6), (7) give the usual Friedmann equations [10,65]. However, if instead of the usual entropy ones applies the generalized, non-extensive one, namely Tsallis entropy [23, 26], which is the correct one to be used in non-extensive systems such as large-scale gravitational ones and it is given by

$$S = \frac{A_0}{4G}\left(\frac{A}{A_0}\right)^{\delta}, \tag{8}$$

with $A_0 \equiv \frac{4\pi}{H_0^2}$, H_0 a constant introduced for dimensional reasons and δ the non-extensive exponent, then (5), (6), (7) lead to [29,30]

$$\delta\left(\frac{H_0^2}{H^2}\right)^{\delta-1}\dot{H} = -4\pi G\left(\rho + p\right). \tag{9}$$

Thus, inserting (4) and integrating we obtain

$$\frac{\delta}{2-\delta}H_0^2\left(\frac{H^2}{H_0^2}\right)^{2-\delta} = \frac{8\pi G}{3}\rho + \frac{\Lambda}{3}, \tag{10}$$

where the cosmological constant Λ appears as a constant of integration. Equations (9), (10) are the two Friedmann equations of modified cosmology through non-extensive horizon thermodynamics [29,30]. Note that in the standard case of $\delta = 1$ they become the standard Friedmann equations.

In the present work we show that the above equations can lead to usual cosmology however with the incorporation of fluids of redefined equation of state. For simplicity we consider the case $\Lambda = 0$ and we re-write Eq. (10) as

$$\frac{3}{8\pi G}H^2 = \tilde{\rho} \equiv C_0\rho^{\frac{1}{2-\delta}}, \tag{11}$$

where

$$C_0 \equiv \frac{3}{8\pi G}\left(4\pi\right)^{\frac{1-\delta}{2-\delta}}\left[\frac{8\pi G\left(2-\delta\right)}{3\delta}\right]^{\frac{1}{2-\delta}}. \tag{12}$$

In the case where the initial perfect fluid has a constant equation of state (EoS) parameter w, namely $p = w\rho$, ρ behaves as $\rho \propto a^{-3(1+w)}$ and therefore $\tilde{\rho}$ behaves as $\tilde{\rho} \propto a^{-\frac{3(1+w)}{2-\delta}}$, which implies that the EoS parameter is effectively changed as

$$w_{\mathrm{eff}} = -1 + \frac{1+w}{2-\delta}. \tag{13}$$

Using the conservation law (4) we acquire

$$\dot{\tilde{\rho}} = -3H\frac{1}{2-\delta}C_0\rho^{-\frac{1-\delta}{2-\delta}}\left(\rho + p\right), \tag{14}$$

and thus we may define the effective pressure $\tilde{p}$ as

$$\tilde{p} \equiv \frac{1}{2-\delta}C_0\rho^{-\frac{1-\delta}{2-\delta}}\left(\rho + p\right) - \tilde{\rho}, \tag{15}$$

in order for $\tilde{\rho}$ and $\tilde{p}$ to satisfy the conservation law $0 = \dot{\tilde{\rho}} + 3H\left(\tilde{\rho} + \tilde{p}\right)$. Hence, (15) can be finally written as

$$\tilde{p} = \left(-1 + \frac{1+w}{2-\delta}\right)\tilde{\rho}\,, \tag{16}$$

which is consistent with (13) as expected. As we observe, through the application of non-extensive thermodynamics in a cosmological framework we were able to obtain standard cosmology but with fluids of redefined equation-of-state parameter.

2.2. Non-extensive thermodynamics from fluids of generalized equation of state

Let us now proceed by considering an inhomogeneous fluid with generalized EoS of the form [61]

$$p = f\left(\rho, H, \dot{H}, \ddot{H}, \cdots\right), \tag{17}$$

where f is a function of ρ, H, $\dot{H}$, $\ddot{H}$, etc. In this case Eqs. (11) and (15) give

$$\begin{aligned}\tilde{p} &= \tilde{f}\left(\tilde{\rho}, H, \dot{H}, \ddot{H}, \cdots\right)\\ &\equiv \frac{C_0}{2-\delta}\left[C_0^{\delta-2}\tilde{\rho}^{2-\delta} + f\left(C_0^{\delta-2}\tilde{\rho}^{2-\delta}, H, \dot{H}, \ddot{H}, \cdots\right)\right]\rho^{-\frac{1-\delta}{2-\delta}} - \tilde{\rho}\,.\end{aligned} \tag{18}$$

Therefore, the non-extensivity of modified cosmology can be absorbed into the redefinition of the general EoS of the cosmological fluid.

As an example we may consider a fluid with the EoS

$$p = g\left(\rho\right) + f_0 H^{\beta}\,, \tag{19}$$

where f_0 and β are constants and g is a function of the energy density ρ. Then Eq. (18) leads to the following EoS:

$$\tilde{p} = \frac{C_0}{2-\delta}\,\rho^{-\frac{1-\delta}{2-\delta}}\left[C_0^{\delta-2}\tilde{\rho}^{2-\delta} + g\left(C_0^{\delta-2}\tilde{\rho}^{2-\delta}\right) + f_0 H^{\beta}\right] - \tilde{\rho}\,. \tag{20}$$

In standard cosmology with a perfect fluid with a constant EoS parameter w (with $w \neq -1$) it is known that

$$H = \frac{2}{3(w+1)t}\,, \tag{21}$$

which implies that $a = a_0 t^{\frac{2}{3(w+1)}}$, and thus $\rho = \rho_0 a^{-3(1+w)}$ gives

$$\rho(t) = \rho_0 a_0^{-3(1+w)} t^{-2}\,, \tag{22}$$

with a_0 and ρ_0 being constants. Then the energy Q in the comoving volume $V = V_0 a^3 = V_0 a^3 t^{\frac{2}{w+1}}$, with a constant V_0, is given by

$$E = -Q = \rho V = \rho_0 V_0 a^{-3w} = \rho_0 V_0 a_0^{-3w} t^{-\frac{2w}{w+1}}\,. \tag{23}$$

Furthermore, applying Hawking temperature (7) using (21) we find

$$T = \frac{1}{3(w+1)\pi t}\,. \tag{24}$$

Hence, inserting the above into the thermodynamical relation (6) gives

$$dS = 12\pi\rho_0 V_0 a_0^{-3w} t^{-\frac{2w}{w+1}} dt\,, \tag{25}$$

which for $w \neq 1$ leads to

$$S = \frac{12\pi(w+1)}{1-w}\rho_0 V_0 a_0^{-3w} t^{-\frac{2w}{w+1}}\,. \tag{26}$$

Hence we resulted to a relation $S \propto V^{-w}$, i.e. we recovered non-extensive thermodynamics. This is the inverse procedure of the previous subsection, and thus it completes the correspondence of fluids with redefined equation-of-state parameter and non-extensive thermodynamics.

3. Extensions

In this section we extend the analysis of the previous section in the case where the non-extensive exponent δ of Tsallis entropy (8) depends on the energy scale, i.e. it presents a running behavior [37]. This arises from the fact that the entropy corresponds to physical degrees of freedom, nevertheless the renormalization of a quantum theory implies that the degrees of freedom depend on the scale. In case of gravity, if the spacetime fluctuates at high energy scales then the degrees of freedom may increase, however if gravity becomes topological then the degrees of freedom may decrease, which shows that in general the exponent δ may depend on the scale.

For cosmological considerations the energy scale may be given by the Hubble scale H, and thus δ may depend on H [37]. We parametrize the dependence by using $x \equiv \frac{H_1^2}{H^2}$, with H_1 a parameter that has dimensions identical to H. Then by following the procedure of the previous section, instead of (9) one finds [37]

$$\left\{\delta + \left[\frac{H_1^2}{H^2}\ln\left(\frac{H_1^2}{H^2}\right)\right]\delta'\right\}\left(\frac{H_1^2}{H^2}\right)^{\delta-1}\dot{H} = -4\pi G(\rho+p)\,, \tag{27}$$

where $\delta'(x) \equiv d\delta(x)/dx$. Integrating (27) and using (4) one finds

$$-H_1^2\left\{x^{\delta(x)-2} + 2\int^x dx\, x^{\delta(x)-3}\right\}\Bigg|_{x=\frac{H_1^2}{H^2}} = \frac{8\pi G}{3}\rho + \frac{\Lambda}{3}\,. \tag{28}$$

Equations (27), (28) are the generalized Friedmann equations that arise from non-extensive thermodynamics of varying exponent.

We consider the same general scenario of [37], namely we choose

$$\delta(x) = \frac{\ln\left[c\left(x^{3-n} + \alpha(x) b_2 x^{2-n} + b_1 b_2^2 x^{1-n}\right)\right]}{\ln x}\,, \tag{29}$$

with

$$\alpha(x) \equiv \frac{n(3-n)}{(1-n)^2} + \frac{n^2}{(1+n)(2-n)} b_1 b_2^2 x^{-n-1}\,, \tag{30}$$

and where $c \equiv \left\{\frac{3-n}{(1-n)^2} + \frac{b_1}{1+n}\right\}^{-1} b_2^{n-2}$, with n, b_1, b_2 the model parameters. For the scenario (29), Eq. (28) becomes

$$-f(x)\big|_{x=\frac{H_1^2}{H^2}} H_1^2 = \frac{8\pi G}{3}\rho + \frac{\Lambda}{3}\,, \tag{31}$$

with

$$f(x) \equiv c\left[\left(\frac{3-n}{1-n}\right)x^{1-n} - \left(\frac{2-n}{n}\right)b_2\alpha(x)x^{-n} - \left(\frac{1-n}{1+n}\right)b_1b_2^2x^{-n-1}\right]. \tag{32}$$

Finally, we mention that for the choices $n=2$ and $b_1=b_2=0$ we obtain $\delta(x)=1$, i.e. standard thermodynamics, and in this case Eq. (31) becomes the standard Friedmann equation

$$H^2 = \frac{8\pi G}{3}\rho + \frac{\Lambda}{3}. \tag{33}$$

Let us proceed by showing that the modified cosmology from non-extensive thermodynamics with varying exponent can be re-written as standard cosmology but with fluids with generalized equation-of-state parameter. In particular, Eq. (28) can be rewritten in the standard form

$$\frac{3}{8\pi G}H^2 = \tilde{\rho}, \tag{34}$$

if we define

$$\tilde{\rho} \equiv \frac{3H_1^2}{8\pi G\mathcal{F}^{-1}\left(\frac{8\pi G}{3}\rho + \frac{\Lambda}{3}\right)}, \tag{35}$$

where $\mathcal{F}^{-1}$ is the inverse function of

$$\mathcal{F}(x) \equiv -H_1^2\left\{x^{\delta(x)-2} + 2\int^x dx\, x^{\delta(x)-3}\right\}, \tag{36}$$

or equivalently

$$\rho = \frac{3}{8\pi G}\left[-\frac{\Lambda}{3} + \mathcal{F}\left(\frac{3H_1^2}{8\pi G\tilde{\rho}}\right)\right]. \tag{37}$$

We can now define the effective pressure as

$$\tilde{p} \equiv -\frac{\dot{\tilde{\rho}}}{3H} - \tilde{\rho}, \tag{38}$$

in order for $\tilde{\rho}$ and $\tilde{p}$ to satisfy the conservation law. Hence, the effective fluid acquires a generalized equation-of-state parameter of the form

$$w_{\rm eff} \equiv \frac{\tilde{p}}{\tilde{\rho}} = -1 - \frac{8\pi G}{3}\rho(1+w)\left[\ln\mathcal{F}^{-1}\left(\frac{8\pi G}{3}\rho + \frac{\Lambda}{3}\right)\right]', \tag{39}$$

with $w = p/\rho$ the EoS parameter of the initial fluid and with primes denoting derivative with respect to the argument.

We now proceed by performing the inverse of the above approach. We consider a fluid with a generalized equation of state of the form (17), and in particular with

$$\tilde{p} = \tilde{f}\left(\tilde{\rho}, H, \dot{H}, \ddot{H}, \cdots\right) \equiv -\frac{(8\pi G\tilde{\rho})^2}{9H_1^2\mathcal{F}'\left(\frac{3H_1^2}{8\pi G\tilde{\rho}}\right)}$$
$$\cdot\left\{f\left(\frac{3}{8\pi G}\left[-\frac{\Lambda}{3} + \mathcal{F}\left(\frac{3H_1^2}{8\pi G\tilde{\rho}}\right)\right], H, \dot{H}, \ddot{H}, \cdots\right)\right.$$

$$+\frac{3}{8\pi G}\left[-\frac{\Lambda}{3}+\mathcal{F}\left(\frac{3H_1^2}{8\pi G\tilde{\rho}}\right)\right]\Bigg\}-\tilde{\rho}\,. \tag{40}$$

Hence, one can immediately see that this fluid has an effective EoS parameter of the form (39).

In summary, the result of the previous section holds also in the extended case where the exponent δ presents a varying behavior, namely there is a correspondence between non-extensive thermodynamics and fluids with generalized EoS. This duality is one of the main results of the present work, and it provides a way of explaining the complicated phenomenological forms of the effective fluid EoS parameters that are being broadly used in the literature. Namely, their microphysical origin may lie in the non-extensive thermodynamics of spacetime.

The compelling feature of the scenario at hand is that one can obtain interesting cosmological phenomenology even if the initial fluid is the standard dust matter. For instance, one can see that for the case $w \geq 0$, if $4\pi G\,(1+w)\,\rho\left[\ln\mathcal{F}^{-1}\left(\frac{8\pi G}{3}\rho+\frac{\Lambda}{3}\right)\right]' < 1$ then $w_{\rm eff} < -1/3$ and thus the expansion of the universe is accelerated, while if $\left[\ln\mathcal{F}^{-1}\left(\frac{8\pi G}{3}\rho+\frac{\Lambda}{3}\right)\right]' < 0$ then $w_{\rm eff} < -1$ and the phantom regime is realized. We mention that these conditions may be obtained even in the case where the explicit cosmological constant is set to zero, namely for $\Lambda = 0$, which is an advantage that reveals the capabilities of the scenario at hand, since in this case the universe acceleration results purely from the non-extensivity, or equivalently from the generalized fluid equation of state.

We close this section by focusing on the early universe, namely we apply (29) at the early times where $H \gg H_1$ and therefore $x \ll 1$. We study separately case I where $b_1 \neq 0$ and case II where $b_1 = 0$ and $b_2 \neq 0$. For case I the Friedmann equation becomes

$$cb_1\left(\frac{1-n}{1+n}\right)\left(\frac{H_1^2}{H^2}\right)^{-n}H^2 = \frac{8\pi G}{3}\rho+\frac{\Lambda}{3}\,, \tag{41}$$

while for case II we have

$$(2-n)b_2^{n-1}\left(\frac{H_1^2}{H^2}\right)^{1-n}H^2 = \frac{8\pi G}{3}\rho+\frac{\Lambda}{3}\,. \tag{42}$$

Hence, concerning the energy density of the effective fluid, for case I we find

$$\tilde{\rho} = \frac{3}{8\pi G}\left\{\frac{H_1^{2n}}{cb_1}\left(\frac{1+n}{1-n}\right)\left(\frac{8\pi G}{3}\rho+\frac{\Lambda}{3}\right)\right\}^{\frac{1}{1+n}}\,, \tag{43}$$

while for case II

$$\tilde{\rho} = \frac{3}{8\pi G}\left\{\frac{H_1^{2(n-1)}}{b_2^{n-1}\,(2-n)}\left(\frac{8\pi G}{3}\rho+\frac{\Lambda}{3}\right)\right\}^{\frac{1}{n}}\,. \tag{44}$$

Therefore, using (38) we can calculate the pressure of the effective fluid, which for case I reads as

$$\tilde{p} = \frac{(1+w)\rho}{1+n}\left[\frac{H_1^{2n}}{cb_1}\left(\frac{1+n}{1-n}\right)\right]^{\frac{1}{1+n}}\left(\frac{8\pi G}{3}\rho+\frac{\Lambda}{3}\right)^{-\frac{n}{1+n}}$$
$$-\frac{3}{8\pi G}\left[\frac{H_1^{2n}}{cb_1}\left(\frac{1+n}{1-n}\right)\left(\frac{8\pi G}{3}\rho+\frac{\Lambda}{3}\right)\right]^{\frac{1}{1+n}}\,, \tag{45}$$

while for case II as

$$\tilde{p} = \frac{(1+w)\rho}{n}\left[\frac{H_1^{2(n-1)}}{b_2^{n-1}(2-n)}\right]^{\frac{1}{n}}\left(\frac{8\pi G}{3}\rho + \frac{\Lambda}{3}\right)^{\frac{1-n}{n}} - \frac{3}{8\pi G}\left[\frac{H_1^{2(n-1)}}{b_2^{n-1}(2-n)}\left(\frac{8\pi G}{3}\rho + \frac{\Lambda}{3}\right)\right]^{\frac{1}{n}} . \tag{46}$$

Since we are in the early universe we can neglect the standard matter fluid, and thus the above expressions become

$$\tilde{p} = -\tilde{\rho} = -\frac{3}{8\pi G}\left[\frac{H_1^{2n}}{cb_1}\left(\frac{1+n}{1-n}\right)\left(\frac{\Lambda}{3}\right)\right]^{\frac{1}{1+n}} , \tag{47}$$

for case I, and

$$\tilde{p} = -\tilde{\rho} = -\frac{3}{8\pi G}\left[\frac{H_1^{2(n-1)}}{b_2^{n-1}(2-n)}\left(\frac{\Lambda}{3}\right)\right]^{\frac{1}{n}} , \tag{48}$$

for case II. Hence, as we can see, we can define the effective cosmological constant as

$$\Lambda_{\text{eff}} \equiv 3\left[\frac{(1+n)\Lambda H_1^{2n}}{3(1-n)cb_1}\right]^{\frac{1}{n+1}} \tag{49}$$

for case I, and as

$$\Lambda_{\text{eff}} \equiv 3\left[\frac{\Lambda H_1^{2(n-1)}}{3(2-n)b_2^{n-1}}\right]^{\frac{1}{n}} \tag{50}$$

for case II. Eqs. (49) and (50) indicate that the cosmological constant is effectively screened, which is an advantage since it allows to obtain an inflation realization even if Λ is small enough in order to be consistent with the late-time universe acceleration.

At this point we should add a comment on the relation of the present scenario with the model of inflation in double-screen entropic cosmology [66,67]. In such model one applies the entropic gravity approach, that arises from holographic considerations, and obtains extra terms in the Friedmann equations that depend on higher powers of the energy density. These terms can then drive a successful inflation in which the holographic statistics on the outer screen may lead to a sizable value of the non-linearity parameter [66,67]. Indeed, having in mind the above analysis, one can see that the extra terms that depend on higher powers of the energy density correspond effectively to fluids with generalized equation of state, and thus falling inside the general class of the scenarios of the present work.

In summary, the scenario at hand may simultaneously describe both the inflation era (where $H^2 \sim \left(10^{24}\,\text{eV}\right)^2$), as well as the late-time acceleration epoch (where $H^2 \sim \left(10^{-33}\,\text{eV}\right)^2$), even if the only fluid that is present in the universe is the standard dark matter one. This is a significant advantage and it is one of the main results of this work.

4. Conclusions

In the present work we showed that there is a correspondence between cosmology from non-extensive thermodynamics and cosmology with fluids of redefined and generalized equation of state. In particular, it is well known that one can obtain modified cosmology through the application of thermodynamics in the universe horizon, and this approach has been recently extended through the use of non-extensive entropy. On the other hand it is also known that one can describe the universe phenomenology through the use of effective fluids with generalized equation-of-state parameter of unknown microphysical origin.

We first established the above correspondence in the case of basic non-extensive thermodynamics, showing first that cosmology from non-extensive thermodynamics can result to effective fluids with redefined EoS parameters, and then completing the picture by showing that cosmology with fluids of redefined EoS results to non-extensive thermodynamics.

We proceeded by investigating the extended case of non-extensive thermodynamics of varying exponent, namely when the exponent depends on the scale, which is a more consistent case quantum field theoretically. As we showed, we also established the above correspondence, nevertheless the involved fluids acquire a generalized EoS parameter and not just a redefined one. This duality is one of the main results of the present work, since it provides a way of explaining the complicated phenomenological forms of the effective fluid EoS parameters that are being broadly used in the literature. In particular, their microphysical origin may lie exactly in the non-extensive thermodynamics of spacetime.

Concerning the cosmological behavior, we showed that at late times the effective fluid may drive the universe acceleration even in the absence of an explicit cosmological constant, and even if the initial fluid is the standard dust matter one. Similarly, at early times we obtain an effective cosmological constant which is enhanced through screening, and hence it can drive a successful inflation without spoiling the correct late-time acceleration.

In summary, the established correspondence between cosmology from non-extensive thermodynamics and cosmology with fluids with generalized equation of state provides a theoretical justification of the latter, and hence the cosmological application deserves further investigation.

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