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Strength of the singularities, equation of state and asymptotic expansion in Kaluza–Klein space time

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ABSTRACT

In this paper an explicit cosmological model which allows cosmological singularities are discussed in Kaluza–Klein space time. The generalized power-law and asymptotic expansions of the baro-tropic fluid index ω and equivalently the deceleration parameter q, in terms of cosmic time 't' are considered. Finally, the strength of the found singularities is discussed.

1. Introduction

It is well known that, the singularities are very common problems in general relativity. From the observational data, it is observed that the expansion of our universe is in accelerating way (Riess, 1998; Perlmutter, 1999; Spergel, 2003; 2007). However, these cosmological puzzlings do not absolutely fit to our current theoretical work. Therefore, there are two methods of attempt to amend it. One idea is the modifications of general relativity as the correct theory of gravity (Durrer and Maartens, 2008; Nojiri and Odintsov, 2007a; Starobinsky, 2007; Tsujikawa, 2008; Nojiri and Odintsov, 2007b; 2008; Capozziello, 2009; Bamba, 2009). Also, the other major idea assumes the validity of general relativity and postulates the existence of an exotic component in the content of the universe known as dark energy (Padmanabhan, 2006; Sahni and Starobinsky, 2006).

After the discovery of the expansion of the universe in accelerating way, deeper studies of the phenomenon of the dark energy showed the plethora of new singularities ("exotic" singularities) different from big-bang. It is well known that, the cosmological singularities are a very interesting problem in general relativity. Hawking and Penrose (1970) and Geroch (1968) state that, the primary characteristic of a physical singularity is the beyond of inextensibility of geodesics. However, the nature of geodesics is not sufficient to capture the detailed features of singularities and distinguish physical from un physical ones. Therefore, singularities are classified in terms of strong and weak type (Ellis and Schmidt, 1977; Tipler, 1977a). In a strong singularity, the tidal forces cause complete destruction of objects irrespective of their physical characteristics, whereas a singularity is considered to be weak if the tidal forces are not strong enough to forbid the passage of objects or detectors. In cosmological models, the big-bang singularity is the one example of strong singularity. An example of a weak singularity is the shell crossing singularity in gravitational collapse scenarios where even though curvature invariants diverge, "strong detectors" can pass the external event (Seifert, 1979). Apart from this, firstly, a big-rip associated with the phantom dark energy studied by Caldwell (2002). The cosmological models involve with singularities are discussed by Dabrowski (2003), and further the classification of singularities are discussed by Nojiri (2005) and Bamba (2012).

Nojiri (2005) discussed various properties of singularities in dark energy models including the phantom type fluid. They classify the finite time singularities into four classes in their models. These classifications are: (I) For $t \to t_f$, $a \to \infty$, $\rho \to \infty$ and $|p| \to \infty$, this is called type-I ("Big Rip") singularity (II) For $t \to t_f$, $a \to a_f$, $\rho \to \rho_f$ and $|p| \to \infty$, this is called type-II ("sudden") singularity (III) For $t \to t_f$, $a \to a_f$, $\rho \to \infty$ and $|p| \to \infty$, this is called type-III singularity (IV) For $t \to t_f$, $a \to a_f$, $\rho \to 0$ and $|p| \to 0$ and higher derivatives of the hubble parameter (H) diverse, this is called type-IV singularity.

Afterward, a sudden future singularity or type-II singularity discussed by Barrow et al. (1986), Barrow (2004), Nojiri and Odintsov (2004), Barrow and Tsagas (2005), Barrow et al. (2010) and Barrow and Graham (2015). Nevertheless, the singularities which fall outside this classification (Kiefer, 2010) are curvature singularity with respect to a parallel propagated basis, which show up as directional singularities (Fernandez-Jambrina, 2007) and also intensively studied recently: the little-rip singularities (Frampton, 2011), and the pseudo-rip singularities (Frampton, 2012b).

Dabrowski and Denkieiwez (2010) discussed various type of exotic (nonâstandard) singularities in their models such as: type-I, type-II, type-III, type-IV and a w-singularity. All the above singularities are characterized by violation of all, some or none of the energy conditions

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which results in a blow-up of all or some of the appropriate physical quantities such as: the scale factor, the energy density, the pressure, and the baro-tropic index . By applying the observational data, they showed that, the models involving exotic singularities may serve as dark energy. In particular, they showed that, some of these exotic singularities may occur in the near future of the universe. There are three energy conditions: the null ($\rho c^2 + p \geq 0$), weak ($\rho c^2 \geq 0$ and $\rho c^2 + p \geq 0$), strong ($\rho c^2 + p \geq 0$ and $\rho c^2 + 3p \geq 0$), and dominant energy ($\rho c^2 \geq 0$, $-\rho c^2 \leq p \leq \rho c^2$), where c is the speed of light, ρ is the energy density, and p is the pressure.

The Kaluza–Klein theory (Kaluza, 1921; Klein, 1926) is useful to unify all the forces under one fundamental law. The basic idea was to postulate one extra dimension to unify gravity and electromagnetism in a theory which is essentially five dimensional general relativity. Many researchers have been worked in this theory, who have found models for various phenomenon in particle physics and cosmology using five (5*D*) or more dimensions (De Sabbatta and Schmutzer, 1983; Lee, 1984; Appelquist et al., 1987; Collins et al., 1989). the cosmological and astrophysical implications of extra-dimension have been discussed by Overduin and Wesson (1997). Subsequently, many authors have studied Kaluza-Klein cosmological models from different aspects (Fukui, 1993; de Leon, 1988; Chi, 1990; Lui and Wesson, 1994; Coley, 1994; Mohanty and Samanta, 2009; Samanta and Dhal, 2013; Samanta et al., 2014; 2017). Recently, Samanta (2014) discussed future singularity of the dark energy cosmological model in Kaluza–Klein space time.

Keeping with the view of the above discussion our work is to look at the classification of singularities involve with Kaluza–Klein cosmological model in general relativity.

2. Equations of motion, solutions and singularities

The metric representation of the Kaluza–Klein space time (Ozel, 2010) is written as

$$ds^2 = dt^2 - R^2(t)\left(\frac{dr^2}{1-kr^2} + r^2(d\theta^2 + \sin^2\theta d\phi^2) + (1-kr^2)d\psi^2\right) \quad (1)$$

where $R(t)$ is the scale factor. There are only three distinct possibilities for the geometry, namely $k = -1, 0, 1$ corresponding to the open, flat and closed model of the universe respectively. The source of the gravitational field is assumed to be perfect fluid which is defined by

$$T_{\mu\nu} = (p+\rho)u_\mu u_\nu - g_{\mu\nu}p, \qquad (\mu, \nu = 0, 1, 2, 3, 4) \quad (2)$$

where u_μ is the five velocity vector, satisfying $u_\mu u^\mu = 1$. The Einstein field equations can be written as

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = T_{\mu\nu}. \quad (3)$$

Here, the units to be considered as $c = 1 = 8\pi G$. Using Eqs. (1) and (2) in (3), it follows that

$$6\left(\frac{\dot R}{R}\right)^2 + 6\frac{k}{R^2} = \rho, \quad (4)$$

$$-3\frac{\ddot R}{R} - 3\left(\frac{\dot R}{R}\right)^2 - 3\frac{k}{R^2} = p. \quad (5)$$

In this paper, we focus on flat model i. e. $k = 0$. The overhead dot stands for ordinary derivative with respect to time co-ordinate. Dividing (5) by (4), we get

$$\frac{p}{\rho} = -\frac{R\ddot R}{2\dot R^2} - \frac{1}{2}. \quad (6)$$

Let us now define the deceleration parameter

$$q(t) = -\frac{R\ddot R}{\dot R^2}. \quad (7)$$

Now, the time dependent baro-tropic fluid index $\omega(t)$ can be defined as the ratio of the pressure and the energy density of the universe and which can be written as

$$\omega(t) = \frac{p}{\rho} = \frac{q(t)-1}{2}. \quad (8)$$

Let us define a non-linear time dependent function $f = \ln(R)$

$$\frac{\ddot f}{\dot f^2} = -2(\omega+1) = -(q+1). \quad (9)$$

From the Eq. (9), we can have

$$\left(\frac{1}{\dot f}\right)^{\cdot} = g(t). \quad (10)$$

where $g(t) = -(q(t)+1)$. Integrating Eq. (10), we get

$$\dot f = \left(\int g(t)dt + k_1\right)^{-1}, \quad (11)$$

which can be solved with two free constants k_1 and k_2,

$$R(t) = \exp\left(\int\left(\int g(t)dt + k_1\right)^{-1}dt + k_2\right). \quad (12)$$

The constant k_2 is the part of a global constant factor $R(t_0) = \exp(k_2)$,

$$R(t) = R(t_0)\exp\left(\int_{t_0}^{t}\left(\int g(t)dt + k_1\right)^{-1}dt\right), \quad (13)$$

models with this type of exponential behavior can be found in Dabrowski and Marosek (2013). Performing the Friedman Eqs. (4) and (5),

$$\rho(t) = 6\left(\int_{t_0}^{t} g(t)dt + k_1\right)^{-2}, \quad (14)$$

$$p(t) = \frac{3(g(t)-2)}{\left(\int_{t_0}^{t} g(t)dt + k_1\right)^2}, \quad (15)$$

where $k_1 = \sqrt{6}\rho(t_0)^{-\frac{1}{2}}$, if ρ is infinity at $t = t_0$, then in this case $k_1 = 0$. Hence, expression of the scale factor reduces to

$$R(t) = \exp\left(\int\frac{dt}{\int g(t)dt}\right), \quad (16)$$

The rate of growth of the function $g(t)$ has several qualitative behaviors. Let us assume the function $g(t)$ has a power series expansion around the point $t = 0$,

$$g(t) = g_0t^{n_0} + g_1t^{n_1} + g_2t^{n_2} + \cdots, \qquad n_0 < n_1 < \cdots \quad (17)$$

The scale factor, energy density and the pressure are obtained as:

$$f(t) = \begin{cases} -\frac{n_0+1}{g_0n_0}t^{-n_0} - \frac{(n_0+1)^2g_1}{(n_1+1)(n_1-2n_0)g_0^2}t^{n_1-2n_0} & \text{if } n_0 \neq -1, 0 \\ -\frac{t}{g_0} - \frac{(g_0+g_1)}{2g_0^2}t^2 & \text{if } n_0 = -1, \quad |t| \leq 2 \\ \frac{\ln t}{g_0} - \frac{g_1}{2(g_0)^2}t & \text{if } n_0 = 0, \end{cases} \quad (18)$$

$$R(t)=\begin{cases}\exp\left(-\frac{(n_0+1)}{g_0 n_0}t^{-n_0}-\frac{g_1(n_0+1)^2}{g_0^2(n_1+1)(n_1-2n_0)}t^{n_1-2n_0}\right) & \text{if } n_0\neq -1,0\\ \exp\left(-\frac{t}{g_0}-\frac{(g_0+g_1)}{2g_0^2}t^2\right) & \text{if } n_0=-1,\ |t|\leq 2\\ \exp\left(\frac{lnt}{g_0}-\frac{g_1}{2g_0^2}t\right) & \text{if } n_0=0\end{cases} \quad (19)$$

$$\rho(t)=\begin{cases}6\frac{(n_0+1)^2}{g_0^2}t^{-2n_0-2}+6\frac{g_1^2(n_0+1)^4}{g_0^4(n_1+1)^2}t^{2n_1-4n_0-2}+12\frac{(n_0+1)^3 g_1}{g_0^3(n_1+1)}t^{n_1-3n_0-2} & \text{if } n_0\neq -1,0\\ 6\frac{g_0^2 g_1^2}{g_0^4}t^2+12\frac{g_0 g_1}{g_0^3}t+\frac{6}{g_0^2} & \text{if } n_0=-1\\ \frac{6}{g_0 t^2}-12\frac{g_1}{2g_0^3 t}+3\frac{g_1^2}{g_0^4} & \text{if } n_0=0\end{cases} \quad (20)$$

and

$$p(t)=\begin{cases}3\frac{(n_0+1)^2}{g_0}t^{-n_0-2}+3\frac{g_1(n_0+1)^2(n_1-2n_0-1)}{g_0^2(n_1+1)}t^{n_1-2n_0-2}-6\frac{(n_0+1)^2}{g_0^2}t^{2n_0-2}\\ \quad -6\frac{g_1^2(n_0+1)^4}{g_0^4(n_1+1)^2}t^{-2n_1-4n_0-2}+12\frac{g_1(n_0+1)^3}{g_0^3(n_1+1)}t^{n_1-3n_0-2} & \text{if } n_0\neq -1,0\\ \frac{3g_0g_1-6}{g_0^2}-\frac{12g_1}{g_0^2}t-\frac{6g_1^2}{g_0^2}t^2 & \text{if } n_0=-1\\ \frac{6g_1}{g_0^3 t}-\frac{3g_0-6}{g_0^2 t^2}-\frac{6g_1^2}{4g_0^2} & \text{if } n_0=0.\end{cases} \quad (21)$$

Please see the behavior of the pressure and energy density with time from the Figs. 1–3 which describes the different phases of the universe. We consider the following five different possible values for the parameter n_0. These singularities are classified in following way (Nojiri, 2005; Dabrowski and Denkieiwez, 2009; Fernandez-Jambrina, 2010):

- For $n_0 < -2$, both pressure (p) and energy density (ρ) vanish at $t = 0$, the scale factor $R(t)$ becomes constant, whereas the baro-tropic fluid index ω diverges. This is called w-type singularity.
- For $n_0 = -2$, the energy density (ρ) vanishes at $t = 0$, the pressure (p) and the scale factor (R) becomes finite, whereas the baro-tropic fluid index ω diverges, which is called special case of generalized sudden singularities or generalized w-singularities (Yurov, 0000) .
- For $n_0 \in (-2, -1]$, the energy density (ρ), the pressure (p), the scale factor (R) and ω all are finite for $t = 0$. Hence, there is no singularities within this range.
- For $n_0 \in (-1, 0]$, the energy density (ρ), the pressure (p) and ω diverge at $t = 0$, whereas the scale factor (R) vanishes. These are called type-III, Big Freeze of finite scale factor singularities.
- For $n_0 > 0$, the energy density (ρ) and the pressure (p) diverges at $t = 0$ as $t^{-2(n_0+1)}$ and the baro-tropic fluid index $\omega \to -1$. This type of singularities can not be entrenched in the classifications of Kaluza (1921) and Klein (1926), since the scale factor does not accept convergent expansions. However, we can notice that, the pressure and the energy density are diverges as a power of time which is different from − 2 and it can be close to t^{-2} for very small enough of n_0. The Big Bang/Crunch and Big Rip have a different value of barotropic fluid index ω at zero, depending on the equation of state, these singularities have the value $\omega = -1$ at $t = 0$ irrespective of the values of exponent n_0. Hence, we may name these, grand rip or grand bang/crunch, depending on the behavior of the scale factor at the singular point.

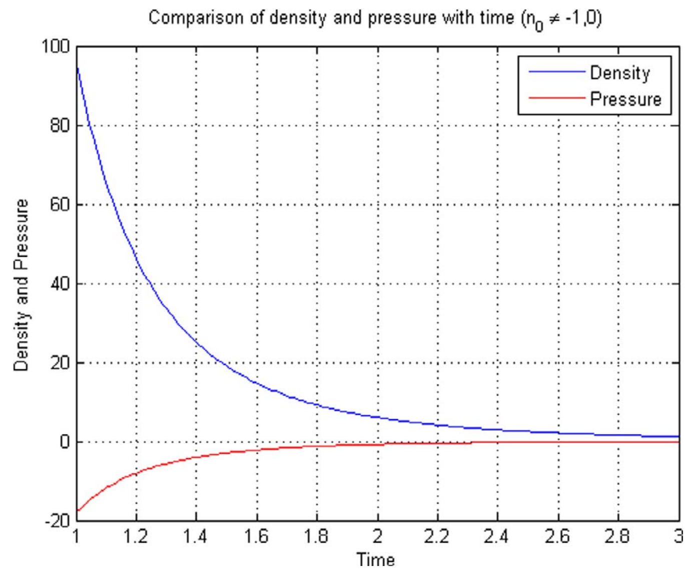

Fig. 1. This figure indicates the variation of energy density and pressure with time. From the figure it is observed that the rapid expansion of the universe has occurred in first phase i. e. in initial epoch which is known as inflationary period of the universe. The present epoch is described by an accelerated expansion phase because of negative pressure.

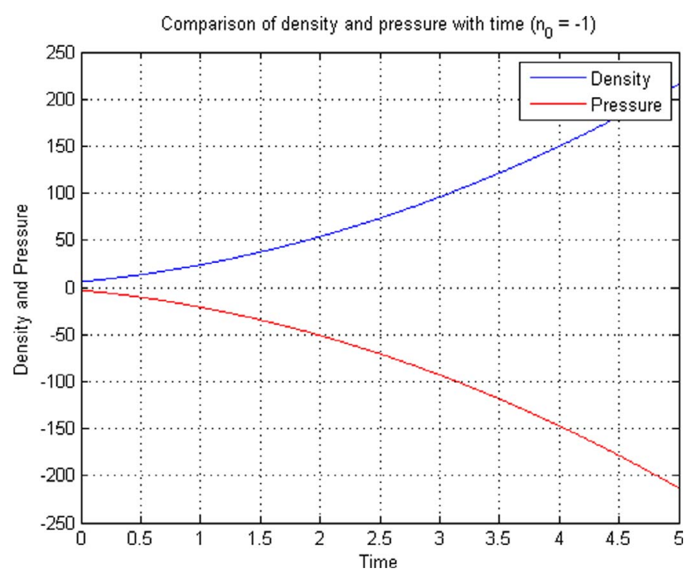

Fig. 2. This figure indicates that the variation of energy density and pressure with time. From this figure it seems that, the universe always characterized by an accelerated expansion i. e. the rate of expansion of the universe increases for ever because the pressure always negative and tends to negative infinity for infinite time. Hence, in this case, there is no deceleration phase.

3. Behavior of the model at infinite time

Apart from the above discussion of the singularities at a finite time 't', we may analyze the behavior of the model at $t \to \infty$. For this observations we can think about the asymptotic behavior of $g(t)$ for large t. If, we take $t_0 \to \infty$, then Eqs. (13)–(15) reduce to

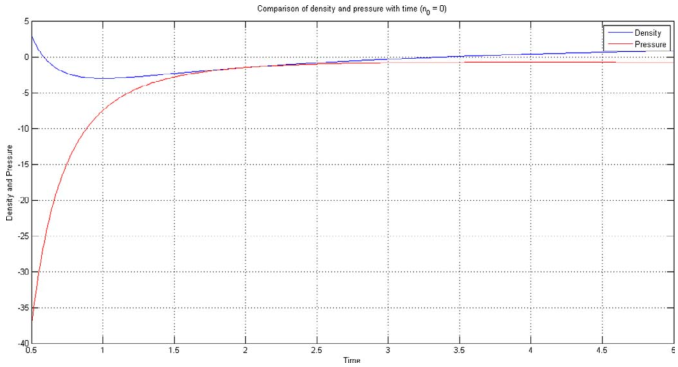

Fig. 3. This figure indicates that the universe is characterized by an inflationary period in initial epoch, decelerated phase in past epoch and accelerated expansion phase in present epoch.

$$R(t) = \exp\left(-\int\left(\int_t^{\infty} g(t)dt + k_1\right)^{-1} dt\right), \tag{22}$$

$$\rho(t) = 6\left(\int_t^{\infty} g(t)dt + k_1\right)^2 \tag{23}$$

and

$$p(t) = \frac{3(g(t) - 2)}{(\int_t^{\infty} g(t)dt + k_1)^2}. \tag{24}$$

If the constant $k_1 = 0$, then the energy density (ρ) and the pressure (p) diverges at $t \to \infty$. The above expressions for scale factor (R), energy density (ρ) and pressure (p) are well define if the integral

$$\int_t^{\infty} g(t)dt \tag{25}$$

is convergent. This ensure that $k_1 = \sqrt{6}(\rho(\infty))^{-\frac{1}{2}}$, which is useful for controlling the asymptotic nature of ρ and p.

Lemma 1. *A necessary and sufficient condition for the convergence of the integral (25), i. e.* $\int_{t_1}^{\infty} g(t)dt$*, where g is positive in* $[t_1, t]$ *is that there exists a positive number K independent of t, such that* $\int_{t_1}^{t} g(t)dt \le K$*, for any* $t \ge t_1$*. The integral* $\int_{t_1}^{\infty} g(t)dt$ *is said to be convergent if* $\int_{t_1}^{t} g(t)dt$ *tends to constant as* $t \to \infty$.

Proof. Since g is positive in $[t_1, t]$ the positive function of t, $\int_{t_1}^{t} gdt$ is monotone increasing as t increases and will therefore tend to a finite limit if and only if it is bounded above. That is, there exist a positive number K, independent of t, such that $\int_{t_1}^{t} g(t)dt \le K$, for every $t \ge t_1$.

If no such type of number K exist, the monotonic increasing function $\int_{t_1}^{t} g(t)dt$ is not bounded above and therefore tends to ∞, as $t \to \infty$ and so $\int_{t_1}^{t} g(t)dt$ diverges to ∞. □

From the above Lemma, we conclude that the integral $\int_t^{\infty} g(t)dt$ is finite, if $g(t)$ is bounded above by $\frac{1}{t}$. This implies that $g(t) \to 0$ for large values of time 't', then from the Eq. (8), we can say that the values of the baro-tropic fluid index ω is − 1. Also, based upon above analysis we can conclude the following points:

- If the integral $\int_t^{\infty} g(t)dt$ is convergent and the value is positive, then we observe that the scale factor (R) from the (19) decreases exponentially to zero as $t \to \infty$. It would be a sort of little crunch. $\omega_{\infty} = -1$ is the asymptotic value of the baro-tropic fluid index ω. At infinity, $R(t)$ is an integrable function, hence this case is included in the set of directional singularities described by Fernandez-Jambrina (2007), which are called strong singularities, but only easy to reach for some observers.
- If the integral $\int_t^{\infty} g(t)dt$ is convergent and the value of the integral is negative, then the scale factor (R) grows up exponentially at infinity. It is the Little Rip (Frampton, 2011; 2012a), or for different types of $g(t)$, the Little Sibling (Bouhmadi-Lopez, 2014).
- For $k_1 \neq 0$, the physical parameters R, ρ and p are obtained from the Eqs. (13)–(15) are well behaved, provided the integral $\int_t^{\infty} g(t)dt$ is infinite. In this case both (ρ) and (p) are tend to zero as $t \to \infty$. The asymptotic value of the baro-tropic fluid index ω_{∞} is − 1 if $g(t) \to 0$.

Now, we may look for the behavior of causal geodesics discussed by Hawking and Ellis (1973). Consider the parameterized curves as $\gamma(\tau) = (t(\tau), r(\tau), \theta(\tau), \phi(\tau), \psi(\tau))$, and impose a normalization condition on the velocity $u(\tau) = \gamma'(\tau)$, depending on its causal type

$$-R^2(t(\tau))(r'^2(\tau) + r^2(\tau)(\phi'^2(\tau) + \sin^2\theta(\tau)\phi'^2(\tau)) + \psi'^2(\tau)) + t'^2(\tau) = \varepsilon,$$

$$\varepsilon = \|\gamma'(\tau)\|^2 = \begin{cases} \text{Timelike:} -1 \\ \text{Lightlike:} 0 \\ \text{Spacelike:} +1, \end{cases} \tag{26}$$

where the overhead dash denotes derivative with respect to the parameter τ.

$$p = u \cdot \partial_r = R^2(t)r'. \tag{27}$$

Eq. (27) together with Eq. (26) permit to make the system of first order differential equations as follows

$$r' = \frac{p}{R^2(t)},\ t' = \sqrt{-\varepsilon + \frac{p^2}{R^2(t)}} \tag{28}$$

for the normal parameter τ.

We analyze to know whether the causal geodesics are complete (Hawking and Ellis, 1973), that is, if the parameter τ can be extended from − ∞ to ∞.

Here, we restrict our discussion to light-like geodesics only:

- **Light-like geodesics**

Since in this case $\varepsilon = 0$, from (28), we have

$$\tau = \frac{1}{p}\int_0^t R(t)dt. \tag{29}$$

Here, $R(t) = e^{-\frac{g_0\alpha}{t^{n_0}}}$, the integral is convergent for positive value of g_0. This implies that, the light-like geodesics meet the singularity at $t = 0$ in a finite normal time τ. Therefore, these geodesics are incomplete. The integral is not convergent for $g_0 \le 0$, and it takes an infinite normal time τ to reach $t = 0$. Therefore, this case yields the light-like geodesics avoid reaching the singularity and are complete in that direction. This is similar to Big-Rip singularities (Fernandez-Jambrina and Lazkoz, 2006).

- **Co-moving time-like geodesics**

If we put $\varepsilon = -1$ and $p = 0$ in Eq. (28), then we can have $t = \tau$ and in both cases geodesics meet the singularity. They are incomplete.

- **Radial time-like geodesics**

Substitute $\varepsilon = -1$ in Eq. (28) and consider $p \neq 0$. Eq. (28) reduces to $t' \simeq \frac{p}{R(t)}$ and $R(t) \ll 1$ for $n_0 > 0$. For $n_0 < 0$, we can have $R(t) \gg 1$ and $t' \simeq 1$ and we found sane conclusions as in the co-moving case, they are also incomplete.

4. Strength of the singularities

Ellis and Schmidt (1977) introduced the idea of strong singularity.

When tidal forces influence a several disruption is called a strong singularity. As per the Tipler (1977b) idea, when volume tends to zero on approaching the singularity along the geodesics is called a strong singularity. Whereas the definition of Krolak (1986) is less restrictive, it is just demands that the derivative of the volume with respect to proper time to be negative. Hence, there are singularities which are strong according to Krolak's definition, but are weak according to Tipler's. Therefore, this definition has been further revised by Rudnicki (2006). From these definition it clear that, $R_{\mu\nu}u^{\mu}u^{\nu}$ is non-negative when an observer moving with velocity u for time-like and light-like events.

- **Light-like geodesics:**

According to Clarke and Krolak (1985) a light-like geodesic meets a strong singularity, according to the judgement of Tipler, the singularity is strong at proper time τ_0 if and only if the integral of the Ricci tensor

$$\int_0^{\tau} d\tau' \int_0^{\tau'} d\tau'' R_{\mu\nu}u^{\mu}u^{\nu} \tag{30}$$

diverges as τ tends to τ_0. According to Krolak's criterion, a strong singularity meet by light-like geodesic at proper time τ_0 if and only if the integral

$$\int_0^{\tau} d\tau' R_{\mu\nu}u^{\mu}u^{\nu} \tag{31}$$

diverges as τ tends to τ_0. The velocity of geodesic is defined as $u=(\dot{t},\dot{r},\dot{\theta},\dot{\phi},\dot{\psi})=\left(\frac{p}{R},\frac{p}{R^2},0,0,0\right)$, integral of

$$R_{\mu\nu}u^{\mu}u^{\nu}d\tau = 3p^2\left(\frac{R'^2}{R^4}-\frac{R''}{R^3}\right)\frac{Rdt}{p} \simeq \frac{3pg_0\alpha n_0(n_0+1)}{t^{n_0+2}}e^{\frac{\alpha g_0}{t^{n_0}}}dt \tag{32}$$

blows up at $t = 0$ for all $g_0 > 0$ and hence these singularities are strong according to both definitions. For $g_0 < 0$ we already know that these geodesics do not even reach the singularity.

- **Time-like geodesics:**

As per the usual definitions, it is worthwhile to know that, the singularities encountered by time-like geodesics are strong or not. According to Tipler's definition, a time-like geodesics meets a strong singularity, at proper time τ_0 if the integral of the Ricci tensor

$$\int_0^{\tau} d\tau' \int_0^{\tau'} d\tau'' R\mu\nu u^{\mu}u^{\nu} \tag{33}$$

blows up as τ tends to τ_0.

Following Krolak's definition, a time-like geodesic meets a strong singularity at proper time τ_0 if the integral

$$\int_0^{\tau} d\tau' R_{\mu\nu}u^{\mu}u^{\nu} \tag{34}$$

blows up on approaching to singularity.

For co-moving geodesics, $u = (1, 0, 0, 0, 0)$, integrals of

$$R_{\mu\nu}u^{\mu}u^{\nu}d\tau = -\frac{6R''}{R}dt \simeq -\frac{6\alpha^2 n_0^2}{t^{2n_0+2}}dt \tag{35}$$

blow up for all $n_0 \geq -2$ and hence singularities are strong at $t = 0$.

For radial geodesics, $u=\left(\sqrt{1+\frac{p^2}{R^2}},\pm\frac{p}{R^2},0,0,0\right)$, the analysis is similar.

$$R_{\mu\nu}u^{\mu}u^{\nu}d\tau = \frac{\frac{-6R''}{R}+3p^2\left(\frac{R'^2}{R^4}-\frac{R''}{R^3}\right)}{\sqrt{1+\frac{p^2}{R^2}}}dt \simeq \begin{cases} \frac{-6R''}{p}+3p\left(\frac{R'^2}{R^3}-\frac{R''}{R^2}\right) & \text{if } R\to 0 \\ \frac{-6R''}{R}+3p^2\left(\frac{R'^2}{R^4}-\frac{R''}{R^3}\right) & \text{if } R\to\infty \end{cases} \tag{36}$$

For $g_0 > 0$; R, R'' tend to zero as $t \to 0$, but the term p is exponentially divergent.

The p term approaches to zero and the integrals of the $\frac{R''}{R}$ term is divergent for $g_0 < 0$. Therefore, radial geodesics meet a strong singularity in both the cases as $t \to 0$. For $g_0 < 0$, singularities are strong for all geodesics except for light-like case, which are not even incomplete.

5. Summary

This work has thus generalized to five dimensional space time the well known results in the four dimensional space time. It is found that there may be significant difference in the principal at least, from the analogous situation in four dimensional space time. Overall, in this paper authors proposed the present behavior of the universe in modified Kaluza–Klein space time within the framework of general relativity and classify some singularity by the help of generalized power and asymptotic expansions of the baro-tropic fluid equation of state of index ω and the deceleration parameter q in terms of cosmic time 't'. We classified the types of singularities into four classes for finite time. The generalized w-type singularities are obtained for for $n_0 < -2$. The special case of generalized sudden singularities or generalized w-singularities are obtained for $n_0 = -2$. For $n_0 \in (-2, -1]$, there is no singularities within this range. The type-III, Big Freeze of finite scale factor singularities are obtained for $n_0 \in (-1, 0]$. The grand rip or grand bang/crunch singularity (it depends on the behavior of the scale factor at the singular point) For $n_0 > 0$ is obtained. At finite normal time $t = 0$, all causal geodesics found incomplete except for light like geodesics at the grand rip, which are incomplete and do not experience the singularity. Finally, we concluded our result with the strength of the singularities and for all geodesics singularities are strong except light-like geodesics.

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