

Development of topological method for calculating current distribution coefficients in complex power networks



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ABSTRACT

The author of this research developed a modified topological method for calculating distribution coefficients of main currents. Main currents are represented as functions of electrical network parameters based on possible trees of directed graphs. Algorithms can be realized in the MATLAB environment. This method can increase load automation of complex electrical networks and systems.

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Introduction

This study deals with the modified method for calculating distribution coefficients of currents (matrix C) in complex electrical networks. The proposed technique is based on a study of analytical expressions derived from the loop current equations and essential current distribution factors, known from the theory of circuits. Analytical study of topological properties of current distribution coefficients in complex electrical networks showed different possible formations of a specific graph tree, without the need to use two trees. Thus, the proposed method significantly improves calculation efficiency of current distribution coefficients used to solve electricity problems. Mathematical expressions with nodal current distribution factors have significant advantages. They do not require bypassing specified circuit routes and simplify common mathematical transformations. The proposed method gives the possibility to form C -matrix setting accurate natural network parameters. This method allows using this matrix as an input in solving problems related to the established modes of electric power systems. The main advantage of the proposed method is significant reduction in the quantity of calculations related to specific trees of a directed graph referring to complex power networks.

Calculations of steady-state regimes of electric power systems are quite complex [1–4]. The difficulty of solving this problem lies in high dimensionality of nonlinear equations describing its composition and mode of telemetry as well as related variety of mathematical models, keeping in mind certain restrictions along with the economic and performance reliability of the system [5,6]. The theory of mathematical models is one of the most difficult and important problems related to the analysis of steady-state regimes of electric power systems. The most attractive among the possible destinations are topological methods. In this regard, development of a mathematical steady state model based on predetermined current distribution coefficients is of particular interest.

In recent years, the problem under study has been considered in many research papers. In particular, the authors would like to note some works [7,8] referring to power flow monitoring, as part of the original approach to using the current distribution coefficient.

In this regard, studying the topological content of current distribution coefficients as a generalized parameter of complex electrical networks is of specific practical and scientific interest.

Problem statement

Calculations referring to complex power systems are significantly simplified by using the known nodal current distribution coefficients (C). The known C -matrix always gives the possibility

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to analyse univocal reaction of electric networks to driving currents. Current distribution coefficients referring to brunch joint resistance adequately characterize specific features of electric networks. In addition, this indicates some universality of their application. C – matrix was studied profoundly and it can be easily found. Distribution coefficient of j node for i branch is expressed as follows:

$$C_{ij} = \frac{I_i}{J_j} \tag{1}$$

It is defined as a complex number, where I_i is a current in i branch. On condition that $J_j = 1$, the equation is expressed as follows [1]:

$$C_{ij} = I_i \tag{2}$$

However, while analysing complex power systems, one may face difficulties associated with excessively large amount of calculations and presentation of relevant results. In this respect, methods based on topology of electric networks are the most attractive. The existing method is complicated by determining numerators of topological expressions through dividing a network into two parts, with the view of finding two trees [2]. Below, the topological method of C-matrix determination is considered. This method is based on using specific features of all possible graph trees, without dividing the network into two parts.

Functional analysis

For the sake of simplicity in describing the proposed research method, current distribution coefficients are presented in the form of analytical functions referring to network parameters. C -matrix elements of the proposed scheme do not depend on scheme parameters. Thus, they can be found directly [1]:

$$C = M^{-1}, \tag{3}$$

where M^{-1} is the first incidence matrix.

In the presence of the closed loops, the C-matrix cannot be found directly according to the scheme or by the formula (3). In the general case, C matrix can be found by the distributed unit current and any known method, for example, the mesh-current method. For circuit presented in Fig. 1, the elements of a column matrix related to distribution coefficients of the driving current are expressed with regard to resistance of branches

$$C = \begin{pmatrix} \frac{Z_2 Z_3}{Z_{11} Z_{22} - Z_2^2} \\ \frac{Z_2 Z_3 - Z_3 Z_{22}}{Z_{11} Z_{22} - Z_2^2} \\ 1 - \frac{Z_3 Z_{22}}{Z_{11} Z_{22} - Z_2^2} \end{pmatrix}, \tag{4}$$

They are defined by solving the following loop-current equations:

$$\begin{cases} Z_{11} \dot{I}_{k1} - Z_2 \dot{I}_{k2} = 0 \\ -Z_2 \dot{I}_{k1} - Z_{22} \dot{I}_{k2} = Z_3 \end{cases} \tag{5}$$

where $z_{11} = z_1 + z_2$, $z_{22} = z_2 + z_3$ are loop resistances.

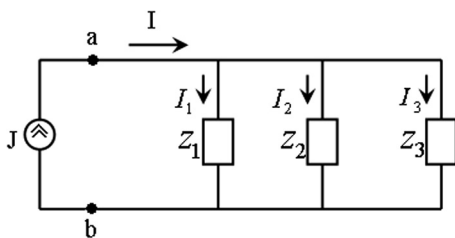


Fig. 1. Typically circuit.

The obtained expressions for the matrix referring to current distribution coefficients are not exponential functions with network topology. Therefore, we provide further transformations of expressions related to matrix (4) by replacing relevant conductivity of branches and after simple transformations, we obtain:

$$C = \begin{pmatrix} \frac{Y_1}{Y_1 + Y_2 + Y_3} \\ \frac{Y_2}{Y_1 + Y_2 + Y_3} \\ \frac{Y_3}{Y_1 + Y_2 + Y_3} \end{pmatrix}, \tag{6}$$

where $Y_1 = \frac{1}{Z_1}$; $Y_2 = \frac{1}{Z_2}$; $Y_3 = \frac{1}{Z_3}$ – are branches conductivity. where $Y_1 = \frac{1}{Z_1}$; $Y_2 = \frac{1}{Z_2}$; $Y_3 = \frac{1}{Z_3}$ express conductivity of branches.

Elements in the matrix column – numerators and denominators (6) expressed through admittance of branches characterize topology of the network, which consists of three graph trees.

Denominators for all coefficients are the same; they are defined as the sum of weight values of trees, and the numerators present tree values including the considered branch.

In the considered case, the tree is characterized by one branch, and its size is defined by the relevant admittance. Generally, weight values of trees are defined by multiplication of branch admittance values [4].

Analysing current distribution coefficients with regard to topology of complex electric networks gives the possibility to determine latent features that could not be provided by using numerical methods [5].

Analytical presentation of distribution coefficients related to nodal currents through their natural parameters, allow overcoming a barrier between intuitive (heuristic) understanding of the problem and its formalization. On the one hand, this is inevitable when using numerical methods. On the other hand, it provides presentation of computational results.

For illustration purposes, current distribution is considered within a scheme, which reflects features of complex electric networks.

Analysis of typical electric network

Let us consider a typical circuit of an electric network shown in Fig. 2.

The first column of the following matrix reflects current distribution coefficients:

$$C = \frac{1}{Z_{11} Z_{22} - Z_5^2} \begin{pmatrix} -(Z_2 Z_{22} + Z_5(Z_3 + Z_4)) & -Z_5(Z_3 + Z_4) & -Z_4 Z_5 \\ Z_1 Z_{22} & -Z_5(Z_3 + Z_4) & -Z_4 Z_5 \\ Z_1 Z_5 & Z_5(Z_1 + Z_2) & -Z_4 Z_{11} \\ Z_1(Z_3 + Z_4) & (Z_1 + Z_2)(Z_3 + Z_4) & Z_3 Z_{11} + Z_5(Z_1 + Z_2) \\ Z_4(Z_1 + Z_2) \end{pmatrix} \tag{7}$$

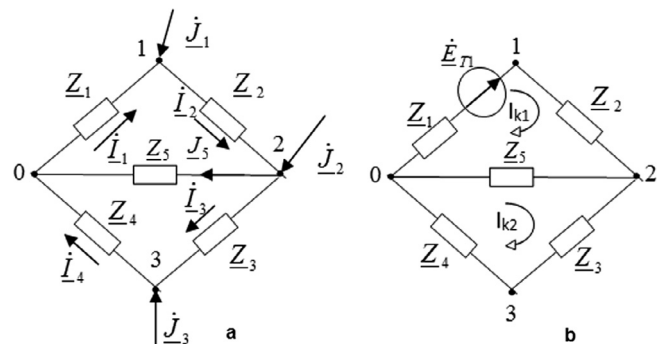


Fig. 2. Typical electric circuit. a) – initial scheme, b) – electric circuit for calculating purposes.

We find them by solving the following loop equations:

$$\begin{cases} Z_{11} \dot{I}_k - Z_5 \dot{I}_{k2} = Z_1 \\ -Z_5 \dot{I}_{k1} - Z_{22} \dot{I}_{k2} = 0 \end{cases} \quad (8)$$

We equate the elements taken from Fig. 2b using the following expressions:

$$\begin{aligned} \dot{I}_{k1} &= \frac{Z_1 Z_{22}}{Z_{11} Z_{22} - Z_5^2} \\ \dot{I}_{k2} &= \frac{Z_1 Z_2}{Z_{11} Z_{22} - Z_5^2}, \end{aligned}$$

where $Z_{11} = Z_1 + Z_2 + Z_5$; $Z_{22} = Z_3 + Z_4 + Z_5$ – loop resistances.

Real currents in circuit branches are determined by elements of the first column (7). Other elements are defined similarly to the described approach by solving loop equations for other assumed circuits, relevant equations are based on Fig. 2a.

Topological formulas of the matrix elements can be defined by transforming the above-mentioned expressions (7) based on replacement of resistance with the relevant admittance. For example, for the first branch related to the first node:

$$C_{11} = \frac{-(Z_2 Z_{22} + Z_5(Z_3 + Z_4))}{Z_{11} Z_{22} - Z_5^2} = -\frac{Y_1 Y_5 Y_4 + Y_1 Y_5 Y_3 + Y_1 Y_3 Y_4 + Y_1 Y_2 Y_3 + Y_1 Y_2 Y_4}{Y_1 Y_5 Y_4 + Y_1 Y_5 Y_3 + Y_1 Y_3 Y_4 + Y_1 Y_2 Y_3 + Y_1 Y_2 Y_4 + Y_2 Y_5 Y_4 + Y_2 Y_3 Y_5 + Y_2 Y_3 Y_4}.$$

It is obvious that denominators present a sum of values of all possible trees, and numerators – the sum of specific graph trees. The value of each tree is characterized by physical features of the equivalent branch admittance. This admittance is transformed in parallel to the circuit's relatively arbitrary node. It follows from another formula related to current distribution coefficients.

The node distribution coefficient implies partial involvement of equivalent branches' admittance in the formation of total admittance of each node relating to the basic node.

The effective calculation method was formulated by analysing current distribution coefficients. In order to ensure clarity and simplicity of the proposed method for determining the topological expression for the current distribution coefficients, we consider the circuit shown in Fig. 2a. Its directed graph is shown in Fig. 3 along with the given admittances of branches (Fig. 4).

Total denominator of all current distribution coefficients is defined as the aggregate value of various undirected graph trees.

$$\begin{aligned} \sum F &= Y_1 Y_5 Y_4 + Y_1 Y_5 Y_3 + Y_1 Y_3 Y_4 + Y_1 Y_2 Y_3 + Y_1 Y_2 Y_4 + Y_2 Y_5 Y_4 \\ &+ Y_2 Y_3 Y_5 + Y_2 Y_3 Y_4 \end{aligned}$$

Obviously, topological expressions of the total denominator obtained by using both methods are similar. Topology of electrical networks provides a certain relationship between current distribution coefficients and the graph structure of a linear network [4]. In this case, the determinant of nodal conduction matrix is equal to the sum of all possible graph trees [3,4]:

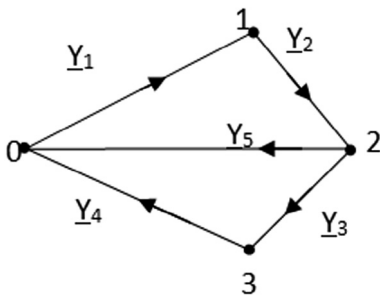


Fig. 3. Directed graph of circuit.

$$\Delta_Y = \sum F,$$

which provides the possibility to verify correctness of the chosen various graph trees. Selection of specific trees gave the possibility to form the numerators of topological expressions. Disturbances in the selected branch are determined by the current distribution ratio.

The directed graph allows determining the signs of values in the numerators referring to the current distribution coefficient. If the branch where one wants to determine the current distribution coefficient of a given node is included in the tree, its value is taken with a plus sign; if a directed graph coincides with the path direction from the node to the base – with a minus sign.

The C_{11} value of the matrix numerator referring to the current distribution coefficient is formed according to the above requirements; it is defined as the sum of the five possible tree values (Fig. 5).

Graph tree elements are calculated according to the formula

$$F_{11} = -(Y_1 Y_4 Y_5 + Y_1 Y_3 Y_5 + Y_1 Y_3 Y_4 + Y_1 Y_2 Y_3 + Y_1 Y_2 Y_4).$$

Consequently, the value of the first element of the matrix is equal to

$$C_{11} = \frac{F_{11}}{\sum F}.$$

It should be noted that the values of specific trees referring to the current distribution coefficient in the numerator are taken with negative signs, since path direction from the first node to the reference one does not match the original circuit branch of this directed graph (Fig. 2).

Thus, one can easily find all remaining matrix elements referring to the current distribution coefficient by using the above method.

Numerators of the remaining matrix elements referring to the current distribution coefficient are determined in terms of the topological approach, respectively.

The second column:

$$\left. \begin{aligned} \sum F_{12} &= -(Y_1 Y_2 Y_3 + Y_1 Y_2 Y_4); \\ \sum F_{22} &= -(Y_2 Y_1 Y_3 + Y_2 Y_1 Y_4); \\ \sum F_{32} &= Y_3 Y_4 Y_2 + Y_3 Y_4 Y_1; \\ \sum F_{42} &= Y_4 Y_3 Y_1 + Y_4 Y_3 Y_2; \\ \sum F_{52} &= Y_5 Y_2 Y_3 + Y_5 Y_1 Y_4 + Y_5 Y_2 Y_4 + Y_5 Y_1 Y_3. \end{aligned} \right\}$$

The third column

$$\left. \begin{aligned} \sum F_{13} &= -Y_1 Y_2 Y_3; \\ \sum F_{23} &= -Y_2 Y_1 Y_3; \\ \sum F_{33} &= -(Y_3 Y_1 Y_2 + Y_3 Y_5 Y_2 + Y_3 Y_5 Y_1); \\ \sum F_{43} &= Y_4 Y_1 Y_5 + Y_4 Y_1 Y_3 + Y_4 Y_2 Y_3 + Y_4 Y_5 Y_2 + Y_4 Y_1 Y_2; \\ \sum F_{53} &= Y_5 Y_2 Y_3 + Y_5 Y_1 Y_3 \end{aligned} \right\}$$

In general, the matrix can be expressed as follows:

$$c = \frac{1}{\sum F} \begin{pmatrix} \sum F_{11} & \sum F_{12} & \sum F_{13} \\ \sum F_{21} & \sum F_{22} & \sum F_{23} \\ \sum F_{31} & \sum F_{32} & \sum F_{33} \\ \sum F_{41} & \sum F_{42} & \sum F_{43} \\ \sum F_{51} & \sum F_{52} & \sum F_{53} \end{pmatrix}$$

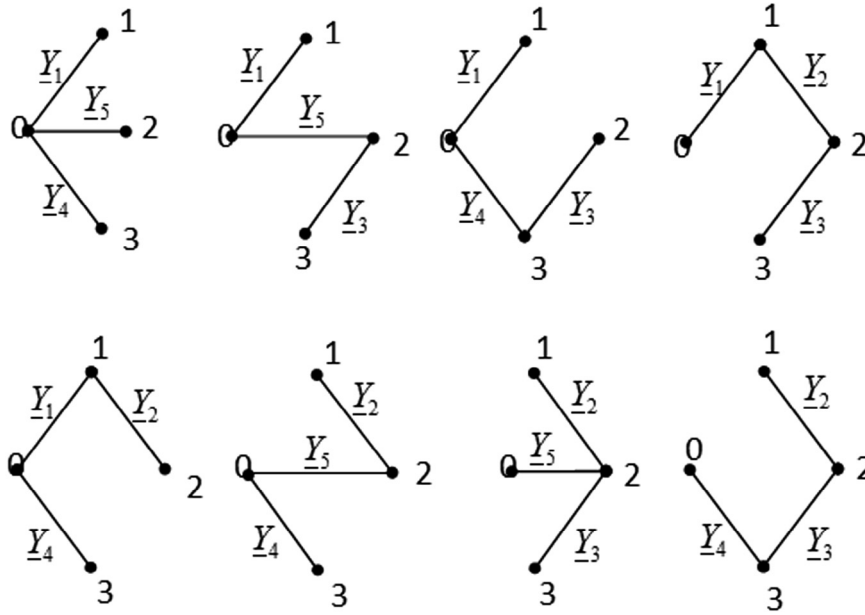


Fig. 4. All possible graph trees.

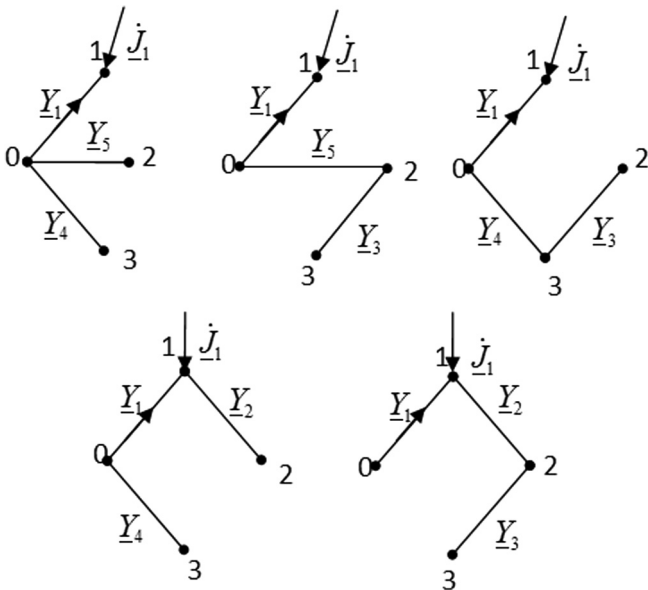


Fig. 5. Possible values of trees.

Analysis of current's distribution coefficients.

All distribution coefficients of main currents are included in the analyzed matrix with the same denominator. Therefore, attention is paid mainly to the topological expression of numerators. Specific tree branches should contain values referring to current distribution coefficients. However, this condition is insufficient. The tree should contain a branch in the path from the first node to the reference one, which is represented by a directed graph. If a directed graph coincides with the path direction from the first node to the reference one, tree values are considered with a positive sign, in other cases – with a negative sign.

The analysis of magnitude and sign of trees referring to the numerator suggests that the current distribution coefficient of the node takes a maximum value in the case of a maximum number of trees with the same sign in the numerator. This makes it pos-

sible to analyse and to determine the most important parameters affecting the modes of electric networks.

Numerical example

In order to determine the formation logic of specific graph trees, a special program in a MATLAB environment was used and calculations were performed in comparison with other well-known techniques. As an example, values were taken from the task [3] and the same results were obtained. Various trees were defined in a MATLAB environment by using the structural theory of numbers [5].

The program for calculating current distribution coefficients is presented in a MATLAB environment. All kinds of graph trees are defined by the structural numbers [3,5].

The graph with circuit parameters [3] is presented in Fig. 6.

Results of calculations are presented below in the matrix form (Table 1).

Numerical values obtained by the above-mentioned methods are similar.

Calculations node voltages existing network

Calculations of node voltages in the existing network of 220 kV are given below in comparison with the calculations performed through the industrial program “RASTR”, used by the dispatching

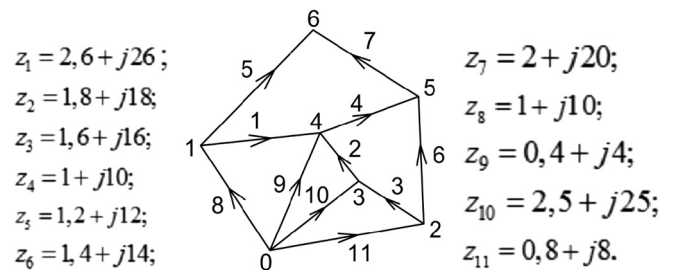


Fig. 6. Graph example.

Table 1
Results of calculations.

lij	1.	2.	3.	4.	5.	6.
1.	0.2085	-0.0093	-0.0297	-0.0740	-0.0076	0.1274
2.	-0.0231	0.0800	0.3525	-0.0872	-0.0184	-0.0213
3.	0.0010	0.1720	-0.3387	-0.0322	0.0449	0.0174
4.	-0.0694	-0.1440	-0.0207	0.1031	-0.5040	-0.2324
5.	0.1477	-0.0525	-0.0306	-0.0279	-0.1637	-0.5941
6.	-0.0782	0.1966	0.0513	-0.0752	-0.3323	-0.1735
7.	-0.1477	0.0525	0.0306	0.0279	0.1637	-0.4059
8.	-0.6439	-0.0618	-0.0603	-0.1019	-0.1713	-0.4667
9.	-0.2548	-0.2148	-0.3435	-0.7357	-0.4780	-0.3385
10.	-0.0241	-0.0920	-0.3088	-0.0550	-0.0633	-0.0388
11.	-0.0773	-0.6314	-0.2874	-0.1074	-0.2874	-0.1561

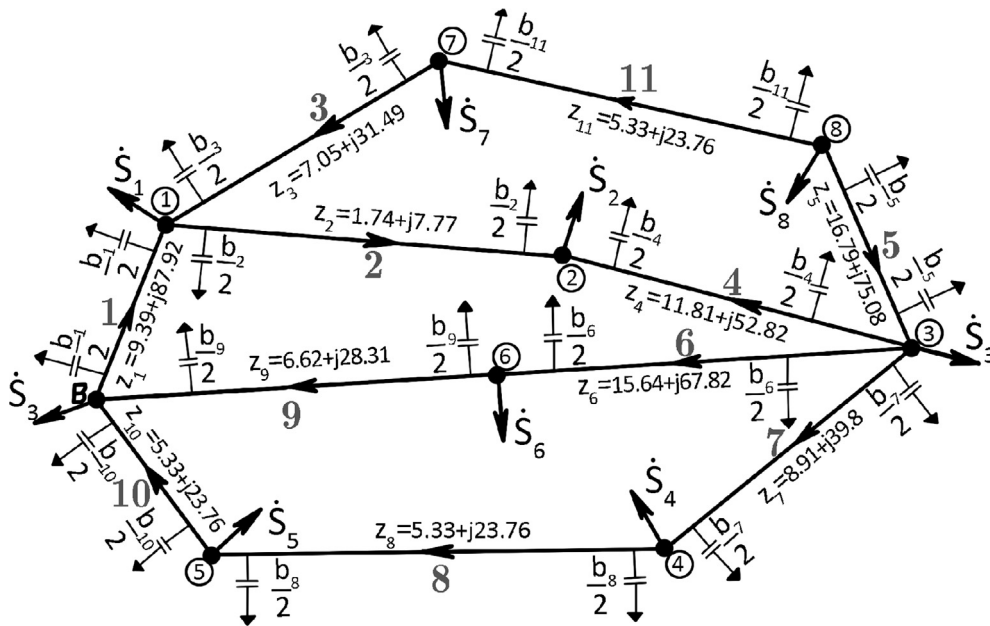


Fig. 7. Example of the existing network.

Table 2a
The values of current distribution coefficients for $I_{ij=1.0.0.4}$.

-0.5043 + 0.0289i	-0.4733 + 0.0272i	-0.2631 + 0.0156i	-0.1432 + 0.0085i
0.3434 + 0.0200i	-0.5958 + 0.0188i	-0.1823 + 0.0108i	-0.0992 + 0.0059i
0.1523 + 0.0089i	0.1224 + 0.0084i	-0.0808 + 0.0048i	-0.0440 + 0.0026i
0.3434 + 0.0200i	0.4042 + 0.0188i	-0.1823 + 0.0108i	-0.0992 + 0.0059i
0.1523 + 0.0089i	0.1224 + 0.0084i	-0.0808 + 0.0048i	-0.0440 + 0.0026i
0.2357 + 0.0146i	0.2504 + 0.0139i	0.3504 + 0.0087i	0.1907 + 0.0047i
0.2600 + 0.0143i	0.2762 + 0.0133i	0.3865 + 0.0069i	-0.3339 + 0.0038i
0.2600 + 0.0143i	0.2762 + 0.0133i	0.3865 + 0.0069i	0.6661 + 0.0038i
0.2357 + 0.0146i	0.2504 + 0.0139i	0.3504 + 0.0087i	0.1907 + 0.0047i
0.2600 + 0.0143i	0.2762 + 0.0133i	0.3865 + 0.0069i	0.6661 + 0.0038i
0.1523 + 0.0089i	0.1224 + 0.0084i	-0.0808 + 0.0048i	-0.0440 + 0.0026i

service “KEGOC”. The single-line diagram of the selected part of the main voltage of 220 kV is presented in Fig. 7.

Calculations of circuit node voltages with parameters were performed according to the formula [4]

$$\hat{U} = U_0 + C^t Z_d \hat{C} \hat{U}_d^{-1} \hat{S}, \tag{9}$$

where

C – rectangular complex matrix of current distribution coefficients;

Z_d – diagonal matrix of branch impedances;

\hat{U}_d – diagonal matrix of nodal voltages conjugate;

\hat{S} – conjugated matrix column nodal loads and generator power.

The resistance data of current distribution coefficients were determined according to branches (Table 2). Cross-capacitive branches were replaced by reactive power and combined with nodal loads.

Design conditions: $U = 230kV, S = P + j(Q_{load} - Q_{line})$.

Table 2bThe values of current distribution coefficients for $I_{ij=5.0.0.8}$.

-0.0716 + 0.0043i	-0.0775 + 0.0048i	-0.4487 + 0.0259i	-0.3956 + 0.0229i
-0.0496 + 0.0029i	-0.0537 + 0.0033i	0.2222 + 0.0179i	0.1066 + 0.0159i
-0.0220 + 0.0013i	-0.0238 + 0.0015i	-0.6709 + 0.0079i	-0.5023 + 0.0070i
-0.0496 + 0.0029i	-0.0537 + 0.0033i	0.2222 + 0.0179i	0.1066 + 0.0159i
-0.0220 + 0.0013i	-0.0238 + 0.0015i	0.3291 + 0.0079i	0.4977 + 0.0070i
0.0954 + 0.0024i	-0.1914 + 0.0030i	0.2622 + 0.0133i	0.2874 + 0.0120i
-0.1669 + 0.0019i	0.1139 + 0.0018i	0.2892 + 0.0126i	0.3170 + 0.0109i
-0.1669 + 0.0019i	0.1139 + 0.0018i	0.2892 + 0.0126i	0.3170 + 0.0109i
0.0954 + 0.0024i	0.8086 + 0.0030i	0.2622 + 0.0133i	0.2874 + 0.0120i
0.8331 + 0.0019i	0.1139 + 0.0018i	0.2892 + 0.0126i	0.3170 + 0.0109i
-0.0220 + 0.0013i	-0.0238 + 0.0015i	0.3291 + 0.0079i	-0.5023 + 0.0070i

Table 3

The values of current distribution coefficients.

Node number	With the use of the matrix C		According to the program RASTR		Discrepancy		
	U, kV	δ , degrees	U, kV	δ , degrees	ΔU , kV	ΔU , %	$\Delta \delta$, degrees
1	241.43	-4.8315	240.65	-4.79	0.78	0.32	-0.04
2	241.53	-4.8433	240.78	-4.81	0.75	0.31	-0.03
3	240.89	-4.3086	240.36	-4.28	0.53	0.22	-0.03
4	237.38	-3.0902	237.1	-3.08	0.28	0.12	-0.01
5	235.60	-1.8466	235.45	-1.84	0.15	0.06	-0.01
6	236.27	-1.9230	236.12	-1.92	0.15	0.06	0.00
7	239.60	-6.2297	238.33	-6.17	1.27	0.53	-0.06
8	239.12	-6.5359	238.03	-6.49	1.09	0.46	-0.05

Node voltage values of the design scheme and the dedicated power supply are presented in [Table 3](#).

The dedicated section of the actual electrical network is, in fact, somewhat busy, and that is confirmed by calculations. As is seen from [Table 3](#), the results obtained by different methods generally coincide. Thus, discrepancy between the voltage calculation results based on using the C-matrix and calculations based on using the program "RASTR" is less than 1.09 kV or less than 0.46%. Discrepancy of the phase component calculations does not exceed 0.06 degrees. It should be noted that real results are similar to those obtained by using the C-matrix. This is determined by the fact that reactive power generated by the line was constant in the calculation process based on the program "RASTR" and refined the calculation of the steady-state voltage, while the proposed method in each iteration was adjusted to the reactive power line. Node voltage values significantly affect reactive power distribution in the line. Active power values in distribution lines largely coincide. In addition, clear simplicity of the mathematical apparatus referring to the proposed method implies (hypothetically) higher speed of software required to use the original algorithm. Therefore, the proposed method is competitive.

Conclusions

We have developed an improved topological method for calculating current distribution coefficients.

The algorithm for calculating values of specific trees in a complex directed graph was developed and implemented in the MATLAB environment with a view to determine current distribution coefficients.

Calculation results of nodal voltages using the C-matrix were almost identical to the data obtained by using the operational program "RASTR" in terms of the existing network of 220 kV.

The authors of this research provided theoretical analysis and demonstrated practical applicability of the proposed approach, tested various options and specified optimal ways to implement the original algorithm, based on its practical use. One should note that the proposed approach can be improved; the authors demonstrated applicability of the principle and possible use of the algorithm, however, current application of the proposed approach is largely theoretical. At the same time, model study of the proposed approach demonstrated accuracy comparable with industrial software. In some cases, when a large proportion of network loads is reactive, the accuracy is even higher. In terms of its practical application, the proposed method can be used to develop new algorithms for relevant industrial software, which will provide greater accuracy for real-world networks and greater productivity. This will result in greater stability of urban power networks, which is in line with the main purpose of this study.

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